University of Alberta

A Study of Researching an Elementary Mathematics Pre-Service Class

by

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Abstract

This thesis explores the author's act of researching the teaching and learning of mathematics and mathematics education. Data collection produced material beginning research that represents one course's duration in one classroom with a teacher and students. By focusing attention towards intentions, various actions are reflected upon as establishing public or private mathematics. For example, patterns established daily re-establish norms during specific episodes. Research questions include processes of designing the study and an identity or role of students. The topic of change concludes this thesis, pointing to how I understand and know *polished math as pedagogy* directing an experience of mathematics. As illustrated by means of a narrative interlude and coda, this metaphor is not separate from a teacher but includes protected places as if it were separate (beyond awareness). Consequently, this introduces an idea about paradoxes: seemingly contradictory texts until perceived from a different perspective and with a revised awareness.

Dedication

This work is dedicated to all who made it possible,
especially my family.

To Shanley, Isaiah, Abigail, and the-one-I-will-meet-soon:
please accept my deepest thanks.

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My name printed on this thesis indicates I have changed. A number of people are responsible for assisting me with this work. For this I am grateful and they need acknowledging with thanks.

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Chapter 1: Introduction

Having graduated from the University of Alberta as a secondary teacher majoring in biology and minoring in mathematics, I started teaching mathematics in a secondary school. As I continued learning about the profession during my first couple of years, I began noticing problematic discrepancies between what I intended to occur and what happened in my classroom. This gap, between intention and action, provides the broad frame for my investigation. Teachers have many pedagogic intentions and students interpret these through the teacher's actions (vocal and otherwise) in various contexts.

During these years early in my career I attended and presented at curriculum meetings. Secondary teachers would present secondary mathematics to others. The climate of these meetings seemed unmistakable: elementary and middle school mathematics teachers talking about their successful curricula and programs and secondary teachers wondering about the functional skills of their secondary students. Being given topics like grade ten rational expressions to present and noticing the effect and affect of reading and writing mathematical notation, my thoughts turned to ideas like differentiated instruction, notation, and what layers occur and happen as people read, write, and talk *mathematics*. Although I did not tend towards differentiated instruction, but more towards layered processes within grade-school mathematics, these meetings began to focus my attention towards elementary school curriculum.

In many ways, an exploration of elementary mathematics from a secondary perspective might direct me towards a different understanding of mathematics and mathematics education: the same questions known differently. By emphasizing some elementary concepts, secondary-level questions might become a series of experiences instead of simply sequential steps to be repeated. By exploring some questions popular at an elementary curriculum level, school mathematics might become an issue of fluency. This layers mathematics and mathematics education by communication (such as reading, writing, and talking).

I returned to the University of Alberta to pursue a Masters degree in mathematics education. I accepted an opportunity to teach an elementary mathematics pre-service methods class (EDEL 316) in Northern Alberta. This was, for me, an apparently new teaching context. Yet, I began noticing similarities in the tensions among my intentions (both mathematical and pedagogic), classroom actions, and modified responses (due to students' actions). Although my previous teaching experience was secondary school mathematics, I decided I could study this new setting with interest and potential.

As I prepared for this course, I discovered one particular feature of the pre-service setting that made it distinctive. Because students intend to be classroom teachers, the teaching itself can be a regular and explicit focus of class attention. The teacher could attempt to make apparent or public his or her intentions with regard to tasks, planning, or other teacher actions. Perhaps naively, I thought this feature would make my study easier: the teacher whose practice I would study would likely

make these usually private intentions more public. As a consequence of such a study, I would enhance my own practice.

In addition to classroom observations, I intended to meet with the teacher regularly. We would explore her intentions, as best we could, and how she saw these playing out in each period (while realizing that these are either before- or after-the-event thoughts and therefore open to some concern as to their authenticity). Knowing about my own attempts at creating a *teachable moment*, and sometimes partially fabricating and verbalizing an intention *in the moment* (Mason, 2002) to bring student attention and awareness to something I wanted them to know, intentions were to become very slippery things.

In the next section, I offer some anecdotes from my own experience of teaching of EDEL 316 as a novice elementary mathematics teacher-educator. Following that, the rest of this thesis will be outlined.

A teaching episode: negotiating many texts

While studying as a graduate student, I accepted an opportunity to teach an Elementary Mathematics Methods course. To prepare, I met with two methods teachers, both with and without their classes, and attended another methods course for an entire semester. I also prepared a curriculum by working, in detail, through the assigned textbook, reading a number of articles in elementary mathematics education (selections from Loewenberg-Ball, Bass, Schifter, Simon, and reviewing *Teaching Children Mathematics*), and examining the Alberta and NCTM curriculum standards.

With fourth-year education undergraduate students, all with previous and successful field experience, I imagined pre-service teachers developing like mathematicians by exploring various methods of reading, interpreting, representing, and doing mathematics – progress they could continue both in themselves and share with future students.

For developing mathematical fluency with elementary curriculum and improving confidence with mathematics, especially when reading elementary mathematics literature (such as textbooks), I chose activities like creating stories, paper-folding, manufacturing manipulatives, and reflecting on various problem solutions. A concurrent study of mathematics education occurred as we worked in groups and as a class remembering ourselves, examining each others' mathematics, and imagining future classrooms and students (with the assistance of an elementary mathematics methods textbook).

As I attempted to explain, expound, describe, illustrate, and *train* less, and use what students shared as points to remember and imagine more, we negotiated issues as a class, including *my success* with communicating *many texts* (Love & Mason, 1992, 35). For example, one lesson started with adding two three-digit numbers with manufactured base-ten manipulatives (the answer not exceeding 1000). Students were instructed to solve a provided question, use base-ten manipulatives and paper to record their steps (using familiar mathematical notation), imagine other possibilities, and attempt to discover and describe different algorithms. I introduced the term "standard algorithm" (beginning with the ones place-value and carrying/trading with superscripts), provided some educated guesses from the research literature about where this specific method might have originated (such as offered in Bass, 2003, provided to students during an earlier period), and gave some reminders about using fingers, benchmarks, rounding, and other mental estimation techniques.

As I walked around the classroom, I assisted many students with how they recorded their actions, which seemed to help them realize their actions were not the same as the "standard algorithm" they were used to, perhaps even *trained by*. When we reconvened as a class, one student made herself and her ideas loud and prominent.

After we began to summarize and share, some students guided this conversation towards teaching the standard algorithm. One student argued anything other than the standard algorithm is difficult and even useless as a progression to other mathematical concepts. My decision to wonder aloud through her conjectures,

using observations from other groups without pointing out specific students, seemed an error. Her voice became louder; she spoke faster until she declared she refused to believe anyone would be able to add these numbers in any other way, especially young elementary students, without first being told how.

With quieter students remaining quiet and louder students frustrated and stopping their mathematics, and needing a transition to another topic, I choose to summarize where many algorithms might start, by illustrating different solutions. "Some of you moved the biggest piece first, perhaps a geometric assumption, and re-started with the phrase, 'that is not the one's piece'". I solved the question multiple times while moving different combinations of pieces (starting with different-sized pieces); I made records with superscripts, sometimes placing these with the answer. Then, I explained how it might be difficult to read these notations; for instance, a reader does not know the author's starting point. One student added to my summary. "Mark, you did that on purpose. You wanted us to confront our own training and work through this problem by counting – without ever telling us the troubles we would encounter." Although I would repeat this intention during different lessons, it would not be until the end of the course when this loud and prominent student apologized with, "I just didn't know what you were doing".

This student, among others, might be described as continually resisting this experience of mathematics education and *that* is my responsibility. (The following 'it' points to a summary topic of this thesis.) Perhaps *it* is an ability for students to be able to function apart from *their teacher's* mathematics. Perhaps *it* is bumping up against multiple algorithms, some difficult to read, which produce the same answer,

and an anticipation of correcting a class of students' work. Perhaps *it* is my attempt at working with *many texts*, when the broad expectation is one that is polished. Perhaps *it* is my actions with layered intentions which I believe lead productively to *many texts*, when these may also lead to mistrusting *one text*.

As is often the case with teachers, one key incident can lead to and fuel a great deal of speculation.

Initial questions

I wanted to immerse myself in researching classroom teaching, especially intentions and interpretations. Researching my own practice did not seem useful at the time. I could also foresee significant ethical dilemmas. Consequently, I chose to explore the teaching and learning in another's classroom. This takes Mason's (2002) book title *Researching your own practice* into a new area: researching my practice by means of researching that of another. Pragmatically, given my most recent teaching experience in EDEL 316 and its challenges – including the relative ease of negotiating access – I chose to explore issues of intention and action in a university elementary mathematics methods class.

In this chapter, I have briefly indicated my interests and chosen research setting. In the next chapter, I discuss some mathematics education literature concerned with elementary teacher education and classroom language research. (Due to the absence of much work specifically focused on the notion of *intention*, particularly with the intentions of teachers, I need to relate this work to my interest with intentions.)

In Chapter 3, I turn to questions of method and describe how my study of this setting unfolded. Chapters 4 to 7 are data chapters and take an ever-closer look at aspects of this elementary mathematics methods class. These descriptions are augmented by excerpts from the lengthy conversations with the teacher. Chapter 4 assumes time as linear indicating aspects of the classroom as a whole. Chapter 5 assumes time as circular (repetitive) focusing specifically on lesson beginnings. Chapter 6 examines a single period from relatively near the beginning of the

semester in greater detail. Chapter 7 examines in fine detail a single incident from one specific lesson: a brief argument between two students initially. By its singularity, the episode re-examines the regularity and orderliness of the whole. Chapter 8 reviews this study in response to my initial questions, including ideas learned.

Chapter 2: Some Research Literature

During my time as a graduate student in the Department of Secondary Education, I read widely, both within mathematics education as a field and methodologically in the area of narrative inquiry. In terms of primary influence and relevance related to this study, there is a small set of authors whom I concentrate on here and in the next chapter: Deborah Loewenberg-Ball (elementary mathematics teacher education), David Pimm (researching classroom language and intention) and Jean Clandinin and Michael Connelly (narrative inquiry which I address in the next chapter). Other people whose work I touch on more lightly include John Mason and Dave Hewitt (see bibliography).

Two articles by Deborah Loewenberg-Ball

Deborah Loewenberg-Ball is an elementary mathematics educator whose work I read in depth, focusing my preparation for teaching an elementary methods course. Her articles "Unlearning to Teach Mathematics" (1988) and "Breaking with Experience in Learning to Teach Mathematics: the Role of a Preservice Methods Course" (1990) highlight attractive ideas I felt drawn towards. Ideas and phrases echo previous conversations with those claiming teaching expertise and echo forward to parts of my research data. (As an example, there is the interview and email exchange with a student, Katrina, examined in Chapter 6, about her perceptions of the role of *student* and *learning from one's teacher*.)

The notion of "Unlearning" in a pre-service classroom inspired my curiosity. Loewenberg-Ball describes a constructivist perspective, pointing to a *learner* as the only identity available in a classroom, and claims that most teachers "are most likely to teach math just as they were taught" (1988, 40). By including and excluding identities, this suggests questions from two identities: learners *acting as* students or learners *acting as* teachers in a pre-service classroom. How influential is Loewenberg-Ball in assisting others to "unlearn" her own influences? How does Loewenberg-Ball exercise this influence? Do pre-service students understand these questions, at some level, and use their responses in negotiating teaching and learning, in *learning to become* a teacher?

Descriptions of her classroom curriculum point to illuminated boundaries for partial answers to these questions. Loewenberg-Ball's descriptions of mathematics, mathematics education, actions, and reflections seem influenced both by effects of "unlearning" and a growth metaphor. With mathematics, students engage "truly as learners". (1988, 41) It seems to point towards students not moving towards Loewenberg-Ball's initiated learning goals.

With actions, she argues (quoting Hawkins) for an environment to "elicit [student] interests and talents". Layering mathematics and mathematics education illustrations, how does Loewenberg-Ball execute this? How else does she assist students who she describes elsewhere as math-phobic and underachieving? (Loewenberg-Ball, 1990) How do students respond appropriately? Finally, with reflections, she encourages student reflections in three directions: towards extending their mathematical understanding; towards attending to feelings and reasons about

persistence and frustration; and towards extending mathematics teaching and learning from grade-school student identities. By further layering these illustrations, reflections seem more a method for classroom management, allowing class movement and change under teacher direction. I return to questions of change in the final chapter.

I experienced further curiosity and wondering about Loewenberg-Ball's curriculum in the quotations she chose from students' experiences. Student evidence contains words such as "just", "so", "never", and "conditioned". Links with the aforementioned seem illustrative towards a ***healing*** more than routines for and about mathematics and mathematics education in a classroom.

"Breaking with Experience in Learning to Teach Mathematics: the Role of a Preservice Methods Course" describes a single methods course; a severe restriction given the variety of routines for and about mathematics and mathematics education in a classroom. [1] Loewenberg-Ball (1990) also introduces this article with her motivation. "As a mathematics educator, my goal is to help my students learn to do something different from – and better than – what they experienced as pupils in mathematics classes." (10) Change to something better re-creates needing a mathematics educator; wonders about her authority, possession, and regard of others; and, wonders about what she considers is better with "a" methods course.

Illustrations of current restrictions and history are layered with a simile and dilemma as Loewenberg-Ball seems to illustrate a place for boundaries and awareness. She writes "prospective teachers can behave like teachers". (11) This seems more about increasing a "learner's" availability for participation in and

synthesizing of curriculum content, curriculum (such as behavioral outcomes), pedagogy, and other meta-actions of a teacher.

She also describes one dilemma from Dewey. Instead of some experiences being mis-educative, it seems all experiences can be educative with adequate meta-evaluation and guidance. Thoughtful initiation requires teacher-student communication about topics of vulnerability – healing, trust, and time. How does Loewenberg-Ball re-create this sort of vulnerability for separating educative from mis-educative experiences during lessons?

While she creates many questions and wonder, she also points to experiences that make mathematics worthwhile (in my estimation). "One of my greatest pleasures of the term was finding a small group of them sitting in the lobby of the College of Education arguing about whether or not 1 was a prime number." (1990, 15) Gripping questions like this one seem necessary for understanding differences between natural and whole numbers and answering more difficult, higher-level mathematics questions. That seems a purpose related to both elementary mathematics and elementary mathematics education.

The literature of David Pimm

David Pimm's continued focus on the nature and role of classroom language in mediating the teaching and learning of mathematics has shaped some of the ideas I attend to, think about, and write about. His 1987 book *Speaking mathematically: Communication in mathematics classrooms* drew my attention, among other things, to the question of pronouns as an element of significance in mathematical classroom discourse (the use of *we* – see also Rowland, 1992, 1999). I draw on this focus in Chapter 7.

His two articles (Pimm, 1992, 1994) on the notion of meta-commenting as a key feature of teaching also attuned me to noticing aspects of this phenomenon, both in terms of my focal teacher's spoken practices as well as what the students picked up on when they were attempting to *be a teacher*. (It has also influenced my own current school teaching practice.) He illustrates this idea in these pieces by means of classroom transcripts providing a number of instances where the teacher's *turn* in a conversation consists of making an observation about one or more previous utterances in that conversation rather than making a substantive contribution to it. Students almost never do this to one another; it is the prerogative of the teacher.

I suggest Pimm has been pointing to intentions and intentionality since the early 1980s, threads of which have echoed in our conversations as well as in his writing, and which have supported my thoughts about intentions and intentionality.[2] Or, perhaps, my interest in this area attuned me to notice particular elements in his writing. Whichever, I use this notion to highlight some aspects in his work.

In "Just a Matter of Definition" (Pimm, 1993a), he reviews Raffaella Borasi's book *Learning mathematics through inquiry*. Here, he does for others' writing what I needed to do for myself: use another's text to explore contemporary pedagogy. He points to differences between a "need to fool" (264) and "mutable" (267). A "need to fool" could interpret ideas of right and wrong for safety and trust: moral respectability. He quotes Jung in an endnote layering perspectives of teacher, teacher-educator, and student while ostensibly pointing elsewhere. "Why does the fact that purposes and intentions change imply that mathematics is *fallible*, rather than merely mutable?" (267; *emphasis in original*). For example, he claims definitions "act as filters through which to view mathematical situations". (270) If a teacher needs 1 to be prime (for reasons of prime factorization), then a definition is worded so 1 is prime. Conversely, if the same teacher does not need 1 to be prime, uniqueness of prime decomposition, then an alternate definition is worded. Mathematics continues.

"There is frequently a proposition concealed behind a definition, verifying that the definition does indeed do what we want it to do. And so, as ever, behind it all are human *intentions*." (271, *emphasis in original*) Additionally, in "Is the Proof in the Practice? A Two-fold Response," Pimm and Jaworski (1990) identify similar intentional processes at work within mathematics.[3] It seems a mathematical text begins and ends within a mathematics context, as a mathematical text. *Fooling* seems a reason and not an end because of mutability.

In using another's text to explore mathematical texts invoking contemporary pedagogy, Pimm points directly to some issues he has with contemporary pedagogy.

Genuine participation with students might suggest the teacher stops meta-processing or meta-commenting for the class. But purposes or intentions are never transparent: "It seems to me plausible that one's purposes may only perhaps be appreciated (if then) at the *end* of a teaching period – so how are we to keep our students with us?" (Pimm, 1993a, 266, *emphasis in original*)

In his second book, *Symbols and meaning in school mathematics* (Pimm, 1996), he offers the notion of *cover story* (89-96) as a form of pedagogically sound fooling, in relation to some algebra teaching practices of Dave Hewitt. In "From Should to Could: Reflections on Possibilities of Mathematics Teacher Education" (Pimm, 1993b), some of the ideas he draws upon, e.g. Mary Boole's writing about teacher-lusts[4], point away from notions of 'better' as a part of change. His use of the term *change-merchants* for some teacher-educator orientations suggests that *change* is misidentified as the be-all and end-all for teacher education (whether pre-service or in-service). "I believe it is dangerous to lose sight of how difficult personal change can be – and we should not talk lightly or glibly about it, let alone expect or demand it." (31) I return to the question of change in the final chapter, where I focus on how carrying out this research has changed me.

The product teachers produce everyday, namely "How I teach", seems inexplicably interwoven with polished mathematics and reasons why it is polished. Pimm uses video technology, attending to some possible moves in order to shift teacher responses to proposals for innovation to seeing possibility, from "should to could". With notions like "illusions" of "speaking accurately", "televisual" or "intimidated responses", "indicating" with "misperceived", "didactic tension", and

"strongly interpretative language", he weaves a fabric of texts towards an event videotaping produces while acknowledging "the event" is not "solely" captured; and, arguably, "*the* event never existed to be captured" (28-29; ***emphasis in original***).

I have tried to bear these philosophical–methodological issues in mind as I worked with my own video and audio recordings. In addition, the work outlined in this section has helped to attune me to possibilities that I would find or did find after the event in my own specific classroom data.

Chapter 3: A Consideration of Method

> I am no soothsayer, unless I hear them, I'll never divine
> Your intentions. (Agamemnon in *Hecuba*)

Claiming to possess a well-developed plan for researching intentions would imply arrogance, at least some, in myself. While intentions and intentionality, as words, are somewhat specified in dictionaries, these words continually point in various ways, to various things. I found no consistent use in the various academic writings I explored: the words were used there, but not specifically attended to as the focus.

My initial research plan was to start simply and arrange to enter another's classroom in order to explore the connection between intention and action, focused on the teacher. As outlined in the first chapter, both in pragmatic terms of ease of negotiating access and in terms of my most recent teaching experience, I opted for an elementary mathematics methods class. Bearing in mind the spirit of the above quotation, I thought I would need to hear intentions expressed, either during classroom periods, or during private conversations with the teacher. I too am no soothsayer.

My methodology is qualitative, an exploration based on a single classroom, like a case study, for an initial collection of research data. After immersing myself in the data, I imagined phenomena of relevance and interest would emerge from repeated exposure (drawing on some orientations from the literature I had read – both before and after the actual data collection). With a naive beginning, ultimately a process of triangulation serves to connect readers, research data, and the author. While the writing here ignores many changes that transpired over the course of

research and writing, I hope this chapter, together with the research literature, will illustrate some of what has taken place.

I needed to open the doors of another classroom with the intention of looking inside towards me. I wanted to enter a classroom and experience another teacher and students. This naive beginning focused my learning about research ethics as a research topic slowly defined itself. For example, one illusion suggests I can read intentions from re-manufactured experiences. Another illusion suggests layered intentions can approach *the experience* and what *is meant*. This potential speaks more about me than anyone else. Thus, the research data, and the subsequent chapters, is more about what I started with and what I continually return to as important.

I found John Mason's (2002) book *A discipline of noticing a*nd Jean Clandinin and Michael Connelly's extensive work on narrative inquiry (see bibliography) provide ideas, illustrations and examples, and actions for methodological organization and negotiation through a time period of research data collection and writing. Mixing Mason's and Clandinin and Connelly's approaches may seem unusual. Mason offers a way of working with my own awareness and writing, while Clandinin and Connelly offer a way with movement and referent ideas.

"Deliberate noticing is much harder than it seems it 'ought' to be. […] Intentions hide prejudices, propensities, habits, and opinions which could be usefully questioned" (Mason, 2002, 37, 42). Mason creates a text for developing research filters that focus attention towards what is not normally attended to. This seems to

occur for him around writing that creates accounts-of and accountings-for; the former strives for remaining as close to un-interpreted description as possible, while the latter additionally attributes cause and intention.

Titling and writing accounts-of focuses words towards describing actions from observations: items that initiate meetings, interviews, discussions, and conversations about various aspects of action. Titling and writing accounting-for focuses words towards justifying actions from observations: items that can be contested since these require a definition. Thus, in the latter, what is not attended include reasons for 'justifying' and 'requiring'. While Mason (and Pimm, 1993b) argues accounts-of pieces should be shared in a brief-but-vivid style, Mason suggests accounting-for pieces be shared within a small, understanding, *vocal*, safe, and secure community enabling hidden intentions to become repeated until the author is aware and able to research these. This creates ways for being at a research site and with participants, and with writing.

For developing attention and perception, Mason suggests cycles and concepts that, with practice, layer into options for choices during data collection and writing. Noticing, marking, and recording focus meta-attention towards attention and actions about memory and memory aides. Researcher actions, while meta-attending to time and context can become issues about communication and exploration. McIntyre (cited in Mason) describes three levels (technical, linking intentions, critical / emancipatory / freedom) to explore different questions about the same event. Metonymy focuses meta-attention towards a "jumping" of thoughts. (Mason, 2002, 78) Inspection (extraspective, interspective, and introspective) focuses meta-

attention towards the location of a researcher relative to others. Finally, meta-attention towards separating routines and habits (including habitual routines) by identifying mechanical, automated, and routine characteristics further develops attention and meta-attention for awareness. As awareness and attention shift, especially with choice, perception changes with increasing amounts of definition.

Clandinin and Connelly offer a text about movement and referent ideas that seem to complement Mason. Parts of narrative inquiry provide words to return to my methodology. "In Narrative Inquiry, it is impossible (or if not impossible, then deliberately self-deceptive) as a researcher to stay silent or to present a kind of perfect, idealized, inquiring, moralizing self." (Clandinin and Connelly, 2000, 62)

Narrative inquiry focuses towards a three-dimensional metaphor and narrative knowing. This metaphor labels dimensions as time, relationship, and place; and one dimension cannot be imagined without the others. Narrative knowing, with a community, expands with a multiplicity of knowing, peripheral senses[5], and wakefulness. Narrative knowing defines lived meanings, while lived meanings seem bounded by the hyphens within co-re-searching: intersubjectivity. For me and this research, intentional awareness requires outside influences.

Methodology applied

In attempting to open a classroom door wider than I could while actively teaching, I needed to negotiate access. Following completion of my ethics application, I contacted an instructor (whom I here call Elizabeth) and explained the focus of my interest and desire to work in her classroom. I subsequently negotiated informed consent with her students (see Appendix 1). In negotiating access to the class, some students gave permission for me to be present, taking notes, make observations and video-tape and audiotape them, but they withheld permission to approach them individually in or outside the classroom, or by email. They made comments like, "I am here for a specific purpose and I don't want to be interrupted by anything else"; "As long as you don't talk to me, I'm fine"; and, "I'm going to hold off signing until I've thought about it." Subsequently, a number of students said my work provided an additional outlet to talk about pedagogy (both in a school and in this classroom) or mathematics.

Thus, I started with an elementary mathematics pre-service course, an instructor, and a classroom possessing descriptions and illustrative episodes of contemporary pedagogy. To become familiar with this course and issues within elementary mathematics (aside from reading/writing/mathematising), I visited other instructors, with and without their classes, experienced the same course with another instructor, and taught the same course as a principal instructor.

The following is a list of research actions I undertook and research data I collected:

- 2000 minutes of classroom time: nearly all audio-taped, 3/5 videotaped;

- records of all classroom handouts;
- review of other resources initiated by instructor;
- review course textbook and instructor's final exam study guide;
- partial and brief review of written assignments from students, with instructor present;
- review of all video tapes during and after course;
- review of all audio tapes after course completion (some during course);
- regular meetings with course instructor;
- meetings and emails with students;
- research journal: notes during classroom time, notes before and after classroom time, notes before and after meetings, personal journal during the course, personal journal after course concludes.

Perhaps the most unusual item of this methodology during the researched course's semester are my suspended judgments about teaching and learning with participants, aside from pointing to intentions, and what occurs in my more private conversations, writing, and reflections.[6] This happened until I began to understand separately from *myself* and begin to admire with reasons and intentions given by others. Sometime after the course's completion, I wrote a journal entry:

> The more I review [the re-manufactured experiences] the more I lose my passionate reasons to explore intentions, the ones that motivate me. The more this happens, the more something else appears about re-manufacturing. I wonder if I can position myself close enough to know.

A Narrative Interlude: A Process of Becoming

This thesis has been written over a long period of time. Before discussing the research data and pointing out some of what I have learned as a result of this study and preparation of this thesis, I must address my responsibility to weave some threads. What I initially intended to do, as illustrated in the consent letters in Appendix 1, did not persist. The more I researched this site and its people, the more I discovered intentions pointing back towards me. This brings into focus an issue of change, as a result of undertaking this work, towards me. In other words, this research points towards me through others.

Chapters one, two, and three, though presented first, were written last. Chapter 7, A Classroom Argument, was the first written. I remember experiencing arguments in a classroom as both a student and a teacher. Making or having an argument are public acts and a move toward more *public* experiences of mathematics continues to be a significant topic of discussion in mathematics education. For many teachers, including myself, *public* involves a significant shift in teaching and learning. My experience of grade-school and post-secondary mathematics was not *public* but *private*. My routines and habits, including habitual routines, both as a teacher and as a student, are primarily intended for more *private* experiences. As a teacher, I found myself attempting *public* (believing in its potential) and evaluating my success as marginal because what was outside my perspective and awareness re-established *private*.

The scale of this research, then, is deliberately chosen. To experience another classroom's teacher and students does not attempt to *know* them. It attempts to *be with* them. Due to this scale, my choices for what to research were immense. Yet, during the classroom argument period, an episode illustrating severe consequences of being *public*, I spread out and focused my researching attention in a number of areas (unlike other episodes). My personal experience whispered I would find my routines and habits, including habitual routines, which seek to re-establish *private*. And I am only able to write this after several years of re-writing and re-telling these intentions.

Narrative inquiry fits well with part of my researcher actions behind the lines of illustration. To write "outside my perspective and awareness re-established *private*" means to protect me. For example, I wrote "suspended judgments" (see Chapter 3). In private and protected conversations, I struggled with questions of criticism during data collection. I was alive to this issue, as a researcher, at the beginning, even in my letter to the teacher. I wrote, "Please note that my purpose is not to offer criticism or possible solutions to issues you or I believe are happening". (See appendix 1) A specific type of criticism exists within me, directed towards others, which serves to protect me. I learnt how to recognize this type, connected to a stubborn emotion and perseverance, and work to change my actions. Hence, and afterwards, I soon began to understand "separately from *myself*" (see Chapter 3).

I needed to tell and re-tell my stories in the context of my research data as I changed. This is significant. Narrative inquiry offers such tools. For example, I brought my old chalk holder to a meeting with the teacher during data collection to

elicit stories about us in a different way. I also acquired keys to the research site and was able to visit it again a couple of years later; physically sitting and standing in places I recognized from data recordings. No one was present and fortunately the classroom furniture and layout had not changed. What and how I remembered influenced my changing and, consequently, my writing.

Narrative inquirers like Clandinin and Connelly (2000) warn about metaphoric soup and attempting to fit research data into one metaphor. I worked through these dangers as I learnt about shifting from creating accountings-for to accounts-of and as I learnt more about *how I describe*. Due to these connections, more likely indicating my misunderstandings, I wove narrative inquiry together with, for example, Mason's *Discipline of Noticing* by my actions. I learnt, through continually making mistakes, how nouns and pronouns shift focus and intended perspective; and how specific verbs, adverbs, adjectives, modal verbs, clauses and phrases, and other linguistic means tend either towards accounts-of or accountings-for. I also learnt, through mistakes, how editing edits a memory as it was in a changed person, another instance of *protecting*. Many times I should have started with a blank page.

What narrative inquiry does not offer is research particular to mathematics classrooms, along with mathematics I can work through. Reading mathematics education research and doing the mathematics within changes a reading of these articles. I need this. Certain phenomena seem particular to these classrooms. And, returning to A Classroom Argument chapter, patterns here, in my experiences,

existed elsewhere to re-establish. David Pimm, along with John Mason and others, assist to point these out as I re-experienced me within my research data.

Chapter 6, Reporting Back, was written next as another example of a *public* activity. This episode is complex, occurs with more minutes, and seems to focus why an extremely rare and brief *public* episode (argument) illustrates what gets re-established. Briefly written, if *public* is how I intend teaching and learning, then structured actions as responsibilities must be known and understood; or, severe consequences will occur. This introduces a notion of control used in both chapters. (It also introduces the metaphor of polished mathematics as pedagogy.)

The word "control" occurs often in my research data. Indeed, as a researcher, I struggled with this as I pointed towards intentions. Therefore, an error is to purely suggest *control* illustrates participants. I have learnt, as a teacher, that my actions function as responsibilities and layer for an experience. Describing my actions as control, or for gaining control, indicates I did not possess specific actions, during specific episodes, I required to morally assume responsibility. These are strong words. For example, when I find myself tending towards *control*, I also remember actions during episodes minutes before (or longer) I did not do and should have. When I write "control", I recall and remember myself in my research data. When I point to specific behaviors of the teacher or students, I do so with the support of the research data and with the responsibility of a researcher looking at them to see myself. And, what I point out does not illustrate them there but me here.

This brings me to Chapters 4 and 5. To further discover *re-establishing*, I slightly change my researcher actions to address larger portions of time and action.

To become patterns, actions must repeat as time progresses linearly: this is Chapter 4. Once patterns and their boundaries seem to form, they influence interpretation and meaning. One consideration of acting as a teacher is recreating a class memory that continues teaching and learning in a spiral, a series of periods and lessons: this is Chapter 5. To summarize research data I coded actions and speech texts from the videotapes (supported with audio-tapes). What I find from the summary of coding are patterns I have struggled with in becoming more *public*.

Collectively, these next four chapters seem illustrative of patterns supporting *public* or *private* action to *be smooth* (and perhaps forgotten) or *be rough*, perhaps *more and remembered*. My purpose, again, is to change me through researching others. As a teacher with more *private* experiences, "*smooth*", "*more*", and "*remembered*" are meaningful to me. I understand, pedagogically, how to do this. If required to act more publically, "*rough*", "*more*", and "*remembered*" are meaningful; and I begin to *protect* myself.

I started in my research site the semester after teaching the same course, both of which took place a number of years ago. At the time and in the course I taught, *protect* was so important to me that I intended to provide an opportunity to students different from anything I had been afforded as a student. I bought a hat and put it on my teacher desk, to be used during conversations to signal times when I would *not* be a teacher. I would sit *with* students like *someone else* and converse in *our* conversation, following social conventions of a peer group, *not* a classroom. At the time, I did not know what would happen; what I would say; what others would say (like the administration); or, what and how students would respond to this learning

opportunity. I did imagine when and how to recognize certain conversations to promote pre-service teachers' vocalized thoughts about *public*, teaching a course for the first time, and about exposing through their teacher what they might encounter in their mathematics classrooms and with students. Perhaps naively, I presumed *public* would be a sign of good faith, to be responded to with good faith. Different students through that semester expressed gratitude and others reminded me why I should be protecting myself. So I firmly set myself to learn different actions (and I illustrated such an episode in Chapter 1).

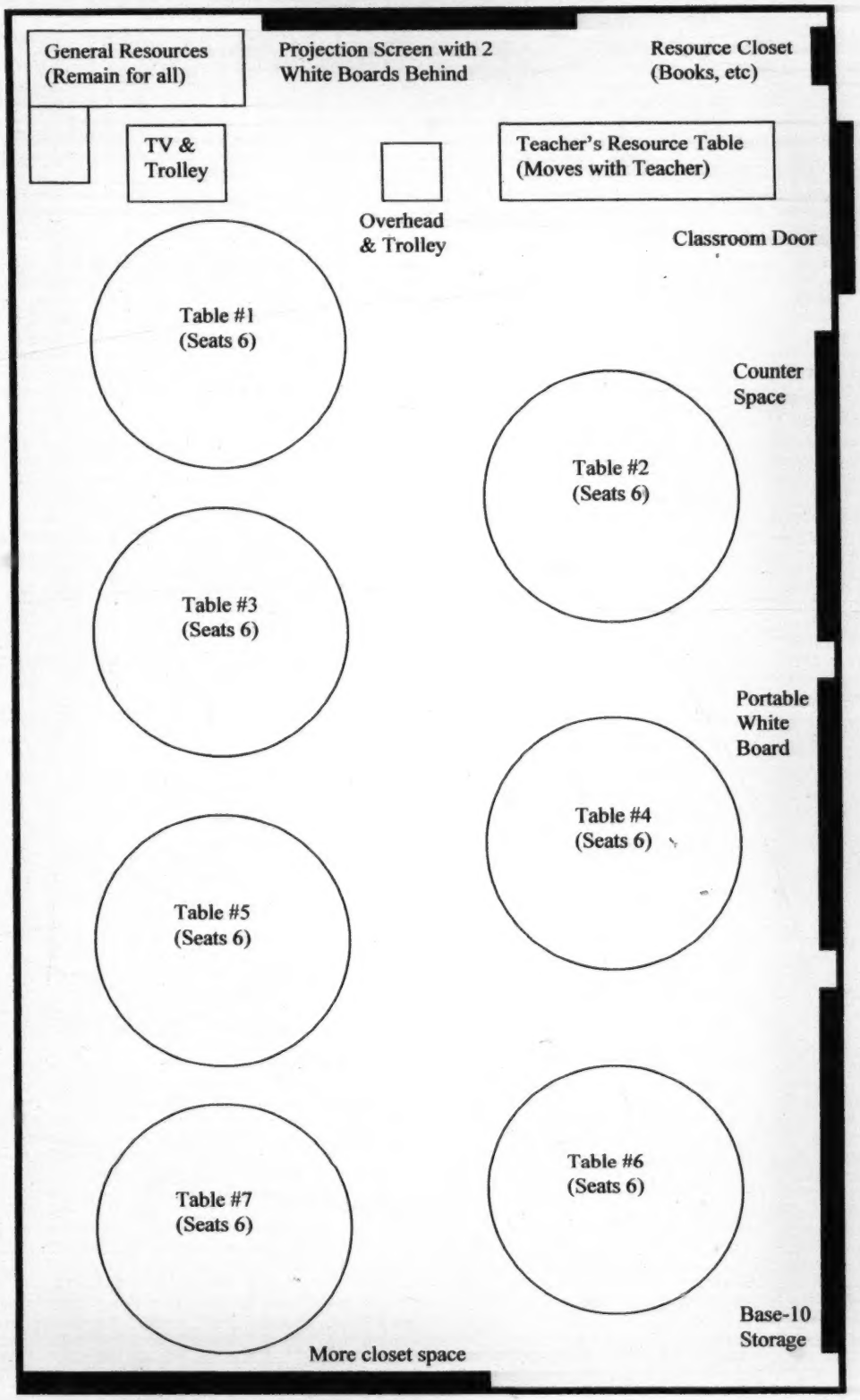
I was sensitive to *protect* and continued being sensitive to it during my research data collection.

Chapter 4: A Classroom Place

Pimm has commented that every classroom has a physical grammar or a set of structures that both enable and limit what is perceived and interpreted. An experienced teacher likely understands this and moulds his or her surroundings to reflect desired intentions for actions as options or choices.

This classroom's furniture layout did not change throughout the semester. (During one ten-minute starter activity during the final week, the first few circular tables were moved.) A sense of place, both for individuals and as a class, seems to develop through this stasis. One possible strength is increasing familiarity. One potential weakness is the notion of an adjustable fit between layout and task. At another level, knowing about this consistent classroom layout enables the reader to imagine events more accurately. Below is an approximate seating chart with an illustration of the classroom furniture (illustration not to scale).

Circular table	Customary number of students
1	4
2	4
3	5
4	5
5	5
6	5
7	2
Approximate Student Total	30


General Resources
(Remain for all)
Projection Screen with 2
White Boards Behind
Resource Closet
(Books, etc)
TV &
Trolley
Overhead
& Trolley
Teacher's Resource Table
(Moves with Teacher)
Classroom Door
Table #1
(Seats 6)
Counter
Space
Table #2
(Seats 6)
Table #3
(Seats 6)
Portable
White
Board
Table #4
(Seats 6)
Table #5
(Seats 6)
Table #6
(Seats 6)
Table #7
(Seats 6)
Base-10
Storage
More closet space

Beginnings and endings

Beginnings and endings bind a course and its periods together temporally. These create and re-create expectations, anticipation, tension and compression, and relationships between people. This section explores some temporal boundaries of the course, the periods, and an illustration of a period's beginning and ending.

The course's periods occur within fourteen weeks, a regular university semester. One week, near the middle, is a university Reading Week with no periods. Twelve periods occur before and thirteen occur after. (One period, immediately following Reading Week, did not occur due to teacher illness.) Two periods occur each week, on Monday and Wednesday. Each period's duration is 80 minutes, scheduled. The teacher extends instructional time for those requesting assistance with regular office hours, twice per week, and provides her home phone number. Finally, a university prepared course survey occurs on the 23rd period.

Beginning all periods, the teacher is already present in the classroom, having taught during the previous one (same course). Entering the classroom, resources are organized on her resource table, a space always full and busy. An overhead projector is often front and center with the light on, slides nearby. A TV-VCR sometimes occupies the same space. Books and other resources regularly line the white-board shelves, counter space, or other physical locations around the classroom. Handouts and evaluated assignments ready for students lie grouped and piled on the side counter. The circular tables, at times, have resources and other items in anticipation of tasks.

Students arriving before the start of the period can pick up handouts and assignments before finding their chairs. The circular tables prove a reinforced social boundary. Rarely did someone move between tables and seating choices are respected. Visiting upon arrival seems as important as finding a chair. Although the teacher does not belong to a particular table, she visits too (each table has space for another person.[7]). This idea of visiting seems to introduce a question concerning actions; to what extent the teacher describes her role as participant or teacher. A participant follows the initiative of the teacher and works at a particular table. A teacher, unable to participate as a student, seems understood as separate; for example, the one-who-stands distinct from those-who-sit. Indeed, being marked as separate is necessary for authority and power with teaching and learning. (Notions of authority and power, as things one gives to another, are explored in later chapters.)

Sometimes this visiting focuses on occurrences or happenings from other classes: conversations about people, content, attention and value, and dynamics of interaction. These are often about things stimulating or frustrating and rarely routine.

Sometimes students would ask something as the teacher walks by; or, they might interrupt her near her resource table. For example, students' might ask, what are we doing today? Responses could summarize forth-coming formal lesson introductions. These examples of repetition, more private discussions repeating more public class discussions, seem a pattern of some students using opportunities addressing their sense of anticipation, prediction, and repetition.

Other questions students might ask define the curriculum-content or define expectations for the period's lesson or the next, bi-weekly, written assignment. For

example, students might ask about how part of a lesson, reading, or experience relates an assignment question. Likewise, the teacher might walk around the classroom and ask about students' assignments at specific tables. Invariably, the formal lesson would begin at some point soon after the planned start time and after the teacher intentionally ended her part in these transitional conversations.

The ending of a lesson occurs in a variety of ways. Sometimes the teacher would stop talking and begin collecting, organizing, and placing her resources on her trolley. Other times she would finish moderating a class discussion about a recent task and begin to focus on the comments of one or two tables or one student, allowing others space to exit. And other times she would provide clean-up instructions and expectations for future periods. Examples of expectations for future lessons are events, such as a video or extra-curricular activity, a task or group of tasks with unique manipulatives, or due-date reminders, like an assignment. After all of these occurrences, students would approach her individually and begin conversations about a lesson or assignment, very similar to the beginning of the period. Thus, the end of a period's lesson is another transitional space between more public and more private instructional time (for students requesting it).

Two markers reinforce the ending of the formal lesson. An auditory marker: increasing noise of students collecting and packing belongings. This noise begins as a gentle rustle and increases to include a loud chorus of many voices and conversations. A visual marker: a few students choosing to stand. Consistently through the semester, within fifteen seconds following a few students standing, the

majority of students would be standing whether the teacher had stopped talking or not.[8]

These ending markers about student actions are not typical markers since these are not expected students' roles or responsibilities. Nevertheless, these begin to illustrate both communication and negotiation of intentions. Individual students' classroom actions relate ideas (exhibited by habits or routines) about knowledge, competence, expectation, and anticipation. By standing and engaging attention-consuming awareness away from the teacher discussion already in progress, these students illustrate generally accepted and unsaid assumptions. Furthermore, many more members of the class demonstrate agreement by following. As a researcher, to attempt a summary about intentions without attempting confirmation in other spaces ignores persistent and consistent patterns about teaching and learning: what I call intentionality. Subsequent chapters will return to these ideas.

Circular and focal areas: front and centre

For the semester, the teacher initiates her formal lessons by walking to the front and center of the classroom and saying, "Good morning". This section explores the location of 'front and center' and starts to examine its possible meaning. From this location, 'being a teacher' and actively attracting attention to initiate authority and power seem to answer which aspects of being a teacher are visible in this classroom. Interestingly, as well as relevant (students are pre-service teachers with field experience and the curriculum is about a duality of curriculum-content; for example, making the teaching transparent), these aspects are unsaid and beyond the usual aspects of age and clothing. (The importance of this place and "teaching" will be revisited in Chapter 6.)

'Front and center' means the teacher is within a three-step radius of the illustrated overhead position: a focal circle. From within this circular zone, the teacher walks, gestures, talks, laughs, and looks at students, sometimes making direct eye contact.

Her voice and position serve both as the beginning of the teacher talk and as a lesson-beginning cue. Therefore, she allows time for students to focus their attention towards her. Her inviting allows students to become participants with her. For example, she sometimes talks about the weather, the season, or part of the semester (like exam week). Then, she might talk generally about the period's lesson, relating it to a chapter title in the textbook. This can include her opinions about the chapter and stories about her classroom experiences. Next, she might talk about administrative items, such as assignments and due dates. Then she might read a

children's book, talk about some related resources, provide organization for the lesson's tasks and activities, or provide organization for future topics and activities. After some time (ranging between one and thirty-five minutes), the teacher introduces a mathematics task or activity and begins talking about it, providing words and thoughts before encouraging students to act through the task or activity.

Sometimes before, sometimes after the students became more active, the teacher initiates a short class reflection. Reflecting-on the activity might include describing the task, talking about her teaching experiences, and inviting students to answer questions about anticipated aspects or characteristics. Questions might involve eliciting participant knowledge connected to the lesson's theme such as, what happened when you were an elementary student. Based on the responses, these questions encourage short, vocal answers, similar to mass media's notion of a sound-bite.

After the task or activity, a reflection often starts with the question, "What happened?" Between one-quarter and one-half of students would contribute comments and answer other questions. Some of these students contribute many comments and answers, others only a few. Also, when the teacher summarizes a task or activity for a closing, she often connects the "what happened in your past" to "a better alternative your students will benefit from". This might be a telling, involve a similar kind of questioning, or both. (When the teacher speaks, she names other class members as students and sometimes as pre-service teachers or teachers. Thus, students might mean children, from the teacher's experiences, or adults and this seems to define teacher.)

This sort of pattern, before and after tasks or activities, describes the general progression of the lesson. Task or activity is connected to the next by means of talking in between. Students are attending to her in this 'front and center' location up to two-thirds of any given lesson.

Circular and focal areas: student tables

When not attending to the teacher's focal circle, students are encouraged to switch their focus to their separate tables. Here, they follow the initiative of the teacher; beginning with her words, actions, memories, remembrances, and with selected students' comments, they engage with the assigned task. Tablemates might choose to work alone, in pairs, or in table groups to interpret the expectations and complete a sequential set of questions. Such table-focused work lasts up to one-quarter of the lesson time.[9]

As the teacher cues a desired attention switch, students start organizing. Using the teacher's guidance and direction (general directions, actions if manipulatives are expected, and some already-perceived-as-frustrating spaces) students would organize their materials, read a handout, or overhead slide for specific directions. If necessary, students retrieve additional materials stored in the classroom. The class is talking and active.

Students might organize roles, within their table according to their group size, by suggesting or volunteering themselves and others. They might narrate personal experiences (similar to the beginning words) or connect to other past situations. Students wonder and ask questions of their tablemates. They expound their understandings. They explain their reasons and evince their knowledge. They might provide answers enabling group movement to a next question or step. Often, and strikingly, these interactions are labeled as "teaching" both by the teacher and many students.

As previously written, the teacher walks around the classroom, from table to table, attempting to engage as many students as possible. "How's it going" is a common question on arrival; though this requires students to interrupt what they are doing and attend to and engage with her. Students would, on occasion, show the teacher something of interest or notice to them. The teacher adds her words and poses further questions for exploration. Students might reveal areas of confusion or frustration, known by lacking a solution, partial proof, or answer. While the teacher frequently answers questions with questions, she would not leave a table without attempting to correct and clarify. Likewise, the students seem to follow the teacher's example about helping and "teaching" their tablemates.

Ignoring speech

In a classroom, voices are attended to for carrying information and comprise a significant majority of content. The next two sections focus on other-than-voice content and the teacher in her focal circle and at student tables (rather than students).

Teacher-talk comprises a majority of both initiating and continuing speech (see Pimm, 1987, for some comparison with classrooms). Reiterating a classroom possesses grammar, some of this talk is talk about talk. These observations illustrate what the teacher is doing, as a teacher, attempting to promote greater awareness for the students about the tasks she is promoting for learning.

Ignoring speech: meta-awareness comments

Pimm (1992, 1994) proposes meta-commenting as a characteristic of mathematics classroom teacher discourse, almost identifying such comments with acts of teaching. Some meta-commenting draws attention to and marks what the teacher is doing, as a perspective, rather than what the student is (or is not) doing or saying. Role modeling, notice my demonstration, and do-as-I-do could be observation-note titles from an observer's perspective. Other meta-comments can promote identities and roles rather than perspectives.

During the semester, many events can be marked as meta-comments highlighting what the teacher is doing. At the beginning of the semester, a different meta-comment undertakes a shifting of identity or role that seems to influence the remainder of the semester. For example, the teacher says at an appropriate time, "Isn't that interesting, I talk and you listen; reflect on that as teachers."

This meta-comment invites students to switch roles and act "as teachers" about what the teacher observes. This suggests the teacher believes students can undertake a shift of role, not simply one of perception. Indeed, this type of meta-comment is one linguistic marker when she refers to students as teachers rather than pre-service teachers.

In a meeting following this period, the teacher reveals she understands this statement as an instance of classroom management and role modeling. A switch of only perception indicates noting a possible reaction and expecting a conscious response after. This seems to allow some classroom content to encourage a feeling, what seems tactile and emotional; perhaps a feeling of empathy for the teacher and responding to it. (Arguably, every teacher experiences a need for waiting-for-the-teacher and immediately wanting others' self-interrupting actions.) Furthermore, this shift is promoted for only a moment since, after this quote, the teacher's voice continues without time or guidance to reflect immediately.

A switch of roles seems to invite and encourage a blurring of words such as teacher, student, teaching, and learning. It ignores actions and responsibilities as defining a title about a person – limits for predicting behavior. Ignoring seems to encourage self-reflective behavior meshing with that of others. Ignoring, within these meta-comments, seems to initiate perspectives emphasizing the teacher.

Ignoring speech: the teacher's "drama"

When the teacher addresses students, she uses what she reveals as "drama".[10] These actions mark a difference between how she behaves and how students behave, with remarkable consistency. In other words, the teacher is the only one consistently using "drama" to attract attention and initiate authority. (This idea of "drama", attracting attention and initiating authority marking a teacher and "teaching", is revisited in later chapters.) Interestingly, this "drama" seems to influence the inclusion and exclusion of words and meanings the class utilizes: another illustration of grammar.

Features that accompany speech are sometimes termed *para-linguistic* and, when these refer to some quality of the actual speech production itself, *prosodic*. The three accompanying elements I will briefly draw attention to are: proximity (para-linguistic), body movement (para-linguistic) and vocal characteristics (prosodic).

The teacher's use of proximity in the classroom correlates with the sort of activity she designs and how she matches it to the class's development. For example, when students switch their focus to their tables, the teacher walks around. Other times, when students focus to the teacher's focal circle, she occasionally steps outside this circle to read from an overhead, allow participants to report back, or encourage students to shift body weight while sitting and attending to her voice. In these ways, students are reminded to direct their attention to the intended circle of focus and the texts of that area. Proximity, while correlating with daily occurrences,

also correlates with semester time. For example, proximity increases before Reading Week.

The teacher's facial expressions, posture, gestures, and laughter all seem to influence the teacher's spoken message, acting as stylistic and punctuation marks. For example, the teacher's facial expressions add ideas like puzzling to rhetorical questions and ridiculous to some described situations. Her postures bring these expressions closer to students, hold expressions, or send these further away. Combining postures with changing facial expressions mark, for example, a paragraph or section breaks.

The dramatization of the teacher's body also changes across semester time. During the beginning periods in the semester the teacher is more dramatic. Later, her dramatizations decrease; due to, perhaps, familiarity.[11] This seems a more direct way of influencing routine responses for power and authority. For example, head nodding and body turning, with stepping, communicates an agreement and shapes an understanding for who contributes. Since the teacher very rarely disagrees publicly with students, a lack of drama might communicate an excluding of texts, perhaps disagreement. Gestures also fit in to these ways of communicating, while attracting attention, as the teacher is the only one to move with "drama" during class discussions.

Chapter 5: Lesson Beginnings

Unlike period time, correlating with clock time, each lesson begins when the teacher chooses. Also, a lesson can begin at any time during a period. Indeed, multiple lessons can occur during one period and lessons might span multiple periods. This chapter explores another text about a grammar of lessons, one structured by a different (more circular) sense of time.

During the semester, most lesson beginnings occur during the first part of the period. Nearly all periods have at least one beginning. Many last about 10 minutes (3 to 35 minutes). Typically each week, one beginning is longer. After a lesson's beginning, a series of tasks and activities occurs, as illustrated with boundaries described in the previous chapter.

For a lesson's beginning, the teacher (Elizabeth) stands in her focal circle and speaks. One important characteristic about this time is she never communicates a plan like other parts of a lesson.

Students seem aware of this. During the first few weeks of the semester, the majority of students attend to her by silently looking on. Afterwards, students seem to choose the direction of their attention. Silence changes to classroom quietness. Some students finish conversations with tablemates. They might hole-punch items or other similar actions. They might write or read. And, for those not initially demonstrating attention, they will demonstrate when invited to be more active or provided cues about the end of this time and the initiating of a lesson's series of tasks and activities.

Another characteristic of 'beginnings' is Elizabeth's rhetorical style. When talking about administrative items and other course concerns, her content is outside a lesson's content. For example, she might talk about important dates or expectations of an assignment. These words detail her specific intentions about demonstration.

Another example of rhetorical style illustrates how Elizabeth might use a lesson's beginning to connect to the lesson's content. Two characteristic quotes, always placed at the start of beginnings, illustrate a belief about time. "Before I start today, I want to …"; or, "I just want to start, though, with …". Then, Elizabeth might talk about an aspect of a classroom that could relate, with hindsight, to the lesson's content. She communicates a separation between what she shares as *being* planned and everything planned.

Elizabeth communicates in various ways about the ending of this time. For example, at the ending of one lesson beginning she says, "Kids usually have a few more wonderings, especially if it means a prolonged break from working". Since a hindsight connects content, when students experience this content, intentions about teacher actions and students actions seem to use a different sight, perhaps in-the-moment-sight. Many actions happening during these beginnings continue as part of student roles. Perhaps, certain feelings about these actions happenings in a lesson's beginning become apart of the lesson's text.

Another example of rhetoric illustrates how Elizabeth talks about promoting thinking. She talks about constructivism opposed to behaviorism, favoring constructivism. Many tasks should be fun, not monotonous; engaging, not repetitive; and valuable, not valued by a grader. She also talks about teachers attending to

student thinking more than what is written: for example, decoding written records for evidence of thinking instead of attending to spelling and other issues of clarity. (This applies both to "her" actual classroom and the ones she imagines aloud for "her" pre-service teachers.) Thinking becomes a concept *to be* understood in the realms of constructivism opposed to behaviorism, contemporary opposed to traditional, and with dichotomies of on-task / off-task, intrinsic / extrinsic, and when a student is thinking / when is he or she not-thinking. In this way, thinking almost becomes tangible, as a feeling.

Intentions moving forward with hindsight: manufacturing

When lessons are about demonstrable outcomes (students' manufacturing something for demonstration), Elizabeth talks about reasons and assumptions for topics, tasks, activities, and questions. For example; a lesson's relationship to assignment questions by relating a series of topics, tasks, and activities using the course's textbook and other resources; and, by relating assignment vocabulary, rubrics, and sometimes what a journey through an assignment might be. For common assessments, Elizabeth talks about due dates, reasons for due dates, reviewing written assignments for expected clarity (before or after completion), illustrating general feedback, providing a final exam study guide, and reviewing this study guide.

For some students, understanding the grader can help create more safety. During this time Elizabeth provides accounts of her own grade-school situations, interesting personal situations, and beneficial student perceptions, interpretations, and summaries. For example, she emphasizes that writing style, skill, and computer processing "will" not be the "only" issues that separate students in assessment, thoughts "will".[12]

Another example of moving intentions forward into a lesson is Elizabeth's use of pronouns. For example, "We do not know the expectations and I hope I can clarify that as we move along". This pronoun switching seems to communicate two different intents. First, and more generally, Elizabeth asserts herself in a variety of groups to exert authority. Examples of these groups (utilizing other examples not

illustrated here) are students, classroom teachers, teacher-educators, mathematicians, professional researchers, published authors, and parents.

Second, these pronouns seem to blur the evaluator's experience. This might be heard as a variety of whispers; for example, a whisper for those who consistently challenge a grader's decisions seeking allowances and a higher grade. These students might appreciate hearing that although Elizabeth writes and intends certain expectations for assessments rubric words are not interpreted objectively. Another whisper might be about learning. For example, some students might appreciate knowing that the lessons Elizabeth intends may not correspond to their interests and she can make exceptions. A blurring (as a metaphor) seems a safe entry point for confrontations without motivating political and group dynamics and actions (such as publicly complaining before addressing issues with Elizabeth). These issues will be addressed in later chapters.

Intentions moving forward with hindsight: elementary mathematical content

Undoubtedly for many pre-service teachers elementary mathematical content creates sympathetic reactions (flight or fight) that must be negotiated.[13] Elizabeth seems to use these parts of lesson beginnings as a negotiation.

Elizabeth reads books, reviews other resources, reviews mathematics education research, or moderates short tasks or activities. These possess some common characteristics. For example, these are promoted as delivering curriculum by encouraging "student thought". These are also self-contained, rich, and delivered in such a way to encourage certain senses.

For any given lesson, any of these resources, tasks, or activities will connect well depending what is emphasized. For example, Elizabeth describes one activity about finding numbers in a newspaper as applicable to all grade levels. During another beginning, she talks about two catalogues and more than thirteen books; her advice, "If it is from the NCTM, you are good to go". She tells about others' mathematics and others' mathematics curriculum. She persuades students with imagined stability and security in delivering what others' develop.

When students are more active during a task or activity, they seem close enough to see and hear mathematics, but not to feel it. The majority of mathematics actions are about recognizing and identifying concepts about number. The majority of mathematics education actions are about recognizing, identifying, and sharing opinions. Since recognizing, identifying, and sharing opinions occur during or after Elizabeth introduces assumptions and beliefs, sharing opinions seem to reinforce the

told assumptions and beliefs. (Students choose sharing by volunteering and, as already mentioned, what is shared repeats Elizabeth.)

As part of this negotiation, humor layers a sense of fun. For example, during a twenty-minute activity of reviewing mathematics research literature, the class focuses on phonetically reading grade one spelling describing addition. Another time, Elizabeth reads *Math curse* (Scieszka & Smith, 1995). She adds comments and questions that release tension, highlighting the quick calculating powers that make many feel inadequate. And, after a mathematics magazine is reviewed for interesting trivia, most students laugh after the following summary. "The next time you are at a party and want to impress people, tell them about Robert Recorde, his book from 1557, and his use of parallel lines to illustrate the idea of equality."

Beginning routines

Lesson beginnings initiate and subsequently reinforce routines for students during a lesson. Elizabeth moderates these portions of a lesson by *being* the teacher. "Students learn about teaching by *being students*."[14]

Regardless of the paper resource, Elizabeth holds it close while talking and turns it showing the relevant section or page. She describes its content with a tone of authority, expertise, and ownership that comes from knowing about many resources. Sometimes, she reads the author's words. On occasion, she pauses and then poses questions about the resource's content, as an elementary teacher might to elementary students. At other times, she waits and then talks about the author's illustrations, the author's writing, mathematics content, or curriculum fit as a teacher-librarian might to pre-service or in-service teachers. Always, before allowing the resource to circulate around the classroom, she reiterates a nature of the work done and describes how an imagined student "constructs" thoughts and enjoys this sort of mathematics.

Some questions Elizabeth asks seem as an elementary teacher to elementary students. An interpretation from actions is students need to be engaged, most thinking and few talking. To encourage thinking, rhetorical questions focus students' thoughts (rhetorical defined with actions such as continuing to speak after posing a question, regardless of wait time). Elizabeth might answer her own question or continue speaking towards a related but different question. Either way, students attend to the details of her question without having the responsibility of responding to the class through sharing answers or clarifying semantics. These sorts of questions

and answers can locate answers within boundaries of right and wrong. (Specific people are never called on to volunteer.)

To engage specific students in discussion, Elizabeth initiates a not-continuing for the class. She might say, often with humor, a comment describing some measurable time like, "I know this is Monday" or "It feels like exam week". She might repeat some key words or phrases of the question, sometimes the entire question. During these comments and repetitions, she uses non-verbal communication like raised eyebrows, leaning forward, gestures, or her physical proximity.

One interpretation after observing a teacher using foresight and designing rhetorical and not-rhetorical questions is providing a design for acknowledging details and encouraging wonder without providing pause enough to engage vocally. (This can also be viewed within areas right and wrong and re-creating safety and security.)

During questions asked as an elementary teacher to elementary students, anticipated answers seem dependent on a quick recall of details. Nearly all answers are limited to short 'sound-bites'; typically one to three seconds long. (Changes between speakers occur quickly.)

Other questions Elizabeth asks seem as a teacher-librarian to pre-service teachers. These reflective questions require opinions, predictions, and justifications addressing an imagined and future class. Two example questions are: "Would you use this resource?" and "Is this resource appropriate for children?" Answers are typically ten to fifteen seconds in length. While these questions seem to address

imagined future classes, they also apply in this classroom; clarifying a definition for *intentional*.

These experiences contain elements of excitement: the pre-service teachers seem excited by motivational solutions. Elizabeth describes this experience to members of this class as "mathematics making more sense to pre-service teachers now than the way you were probably taught".

Chapter 6: Reporting Back

This chapter explores an episode of participants reporting their mathematics back to the class during the fourth period of the semester. In addition to providing more specific detail to some features of the class as a whole, it is particular in two ways. First, it provides greater definition for Elizabeth's declared intent of having students learn to be teachers by acting like teachers. Second, students focusing on their own mathematical problem solving, and for those presenting making their thought and work publicly accessible, is uncommon.

Introducing the mathematics problem

In the previous period, students had watched a video (Burns, 1994) where a Horse Problem had been introduced. The video illustrates two classroom situations by means of interviews and film segments: traditional vs. contemporary mathematics classrooms. Adults relate stories of reporting back during their grade-school periods when they sat in rows. These interviews illustrate these adults' recollections of confusion as younger students; being unsure about teachers' questions; not being sure of solutions (or proofs) or of answers; not sure of their own writing and temporal reflections through that writing; and, not feeling stable, secure, or safe for sharing mathematics publicly.[15]

The horse problem serves as a narrative thread during this video. It reoccurs in a cartoon role-play that includes a talking horse. Many of the interviewed adults are asked to provide an answer. The video does not illustrate these adults' solutions.

In quick montages, many different answers are illustrated (for example, first impressions like laughter and puzzlement). Associated with these images are the adults' occupations. These illustrate a presumed amount of daily arithmetic and mathematical exposure. Thus, the video invites wondering about grade-school stories, contemporary mathematics classroom illustrations, adult occupations, and different answers for the horse problem.

Today, the students focus to Elizabeth's focal circle for an introduction and to recollect the previous period, "OK. So, today we are going to work on problem solving. Yes! Right?" She continues to invite participants to attend to her as her texts become more mathematical. Her enthusiasm invites participation while everyone can read an overhead slide with a cartoon about problem solving. "Well. Let me just see. How many of you, or which of you, really enjoy solving math problems?" Some students turn to their tablemates and chatter fills the classroom.

"How many of you would rather do clean the oven or the fridge or whatever than solve a math problem?" Now, laughter fills the classroom, including Elizabeth's. Participants volunteer a response with raised hands, including Katrina (a student who will be in focus for much of this chapter). Elizabeth continues:

> It's interesting how that works. And you know it is interesting because it is no reflection on your ability. It is probably more of a reflection on how you were taught, and the expectations that were put forth for problem solving. And also then on your confidence for problem solving. It is interesting to me because when I have worked with lots of teachers, and then in these courses with pre-service teachers, so often people think they are not good problem solvers. And you know what, I beg to differ. I watch and, as you have to solve problems for me in your next assignments and explain how you did it, you are marvellous thinkers. What happens is that often we were taught in a context that was very narrow and we were expected to solve things in a particular way that maybe wasn't exactly how we thought. And so, what

> happens then is our confidence in ourselves as problem solvers is kind of shaky. And we think that our way is not the best way or the right way. So those are the things we need to think about now that we are shifting how we do problem solving. And, therefore how we teach it.

She then finds the Horse Problem acetate and places it on the overhead.

Horse Problem

> A man bought a horse for $50.00, sold it for $60.00, bought it back for $70.00 and was considering selling it for $80.00. Would he make money or lose it? How much?

> So, how about that horse problem? You know, from the video? I always think I should be asking the horse because he probably knows the answer. It is so interesting. I have done this problem many many times with teacher groups, in my classes, and also with parent groups, because as you can see with that video, it was set up and done initially for parents, hence the title, "What are you teaching my child?" And it works very well to help to explain to parents and communicate to parents what this looks like and therefore why we are doing some of the things we do in the classroom and that may be different from what … how they were taught. And I find that, you know, most of the time when parents are helped to understand what it is we are doing and why we are doing that they are very supportive of it. It is just that if we do not understand something then, you know, we can feel a little more hesitant. So, with this horse problem I find it very interesting how it can – there is something kind of wiggling about it. There is something like a conundrum that happens in this horse problem that it is kind of confusing. And we can look at it from different ways and if you look at it from different ways we end up getting different solutions. Well, there is one right answer but how do we get there?

As Elizabeth begins to provide instructions, some students are pointing to the overhead and beginning work on the problem. Many have their heads directed to their tables, their paper, and seem to be writing. Few are facing her.

> So, what I want you to do this morning is to take a couple of minutes on your own, in your head, first, and sort this out. And then I want you to just record your solution process. Like kind of how you thought about it, out, yourself, on a piece of paper. OK? Just so you have a chance to sort it out yourself. Then once you have done that I want you at your table to kind of talk about

> your solutions. You might argue a little bit. I think we had a very nice little argument this morning about it and we all didn't come up with the same answer. So, I would like you then to share that [at your tables] and then we will talk about it.

At this point, Elizabeth crosses her focal circular boundary for the first time, walking towards the students in the middle of the class. In the above commentary, she uses the words "argue" and "argument", with the diminutive "little" to downplay, I suggest, the negative force that these related words can have. She also prepares students, I further suggest, *when* they do not "come up with the same answer". She seems to be both sanctioning and valuing a certain amount of disagreement, though she is not explicit about which aspects of "not coming up with the same answer" made it "very nice". As we shall see in the next chapter, when significant public disagreement occurs between two students (the only one I recorded during the entire semester), the teacher's intent there is quite different. (I should also point out that the argument concerns the pedagogic practice of grading children's work rather than a mathematical problem.) Chazan and Ball (1999) refer to a distinction between *making an argument* and *having an argument* and how easily the first can devolve into the second.

Elizabeth then hands over focal attention to individual tables and groups with these words:

> OK, so take about five or six minutes to do that. So, think about it, solve it in a way that makes sense, then, you know, just jot it down so you have your solution down and then you can talk about it. And this [gesturing to the overhead slide] is the exact problem from the video.

Introducing Katrina and others seated with her

Katrina's table is located near the back of the classroom. People who sit here are rarely late. Consistently for the entire semester, all but one student arrive early and visit before the period begins. In many ways, this table is like other tables. Tablemates tend to agree and desire togetherness. However, agreement and togetherness do not mean this table always chooses allegiance first. It seems they experience through their friendship. Each person has and takes opportunity to talk aloud as he or she thinks. Questions and answers are shared. These students seem to believe that their unity enables everyone to accomplish more.

The exchange given below occurs about 30 seconds after the teacher stops addressing the class as a whole. Elizabeth was standing at the back of the classroom, talking with me for a moment (a useful action for not immediately getting involved with a table as students shift from a class focus to an individual or small-group one). Just before the exchange starts, Katrina is recounting her solution and specific answer to the rest of her table.[16]

"He does." (*Katrina*)
"No he doesn't." (*Tablemate*)
"He does. I'm telling you." (*Katrina*)
"No he doesn't." (*Tablemate*)
"We have an argument going on." (*Teacher*)
"Prove it [to the same tablemate]." (*Katrina*)
"Ok so." (*Teacher*)
"Prove it." (*Tablemate*)
"I will." (*Katrina*)
"OK, so you say your thing and then they can explain and then just see. It is so
interesting. [*The teacher immediately walks to another table.*]
"I can explain this." (*Katrina*)

[Katrina's head changes elevation with her phrases as if to punctuate and emphasis her statements. She seems capable of moving her body to affect other's interpretation of her words, like her teacher.]

Katrina's conversation with a tablemate changes from telling to a *does–doesn't* argument. Within a few turns of this interchange, the teacher takes responsibility for both of them. The teacher's meta-comment (Pimm, 1992; 1994, pp. 139-144) "We have an argument going on" is a classic teacher device that achieves two ends simultaneously: both acknowledging and interrupting the argument at the same time. It draws attention to what everybody in some sense already knows, but by so doing it interrupts the conversation by making a shift in the conversational exchange. Pimm (1994) argues the making of meta-comments is one clear marker of "being a teacher". Yet despite this, the next turn is still a peer challenge, "Prove it", which is repeated and Katrina takes up the second challenge with "I will".

Elizabeth offers a second meta-comment to interrupt them. Instead of defending opinions, solutions, and answers, she tries to initiate a process for regaining agreement, by providing a short list of instructions, the rules of engagement. Perhaps these provide stability and security. However, these actions have already occurred. Katrina did have her "say". The others were given space to "explain". Elizabeth seems confident in her way of removing students from this repetitive type of argument. Perhaps she experientially understands these rules of engagement are successful with most participants.

Once Elizabeth leaves, Katrina asserts, "I can explain this". She seems to realise the need for changing from her previous expounding and the explaining. She

chooses words that she will use later when reporting back to the whole class. Katrina answers questions as her tablemates attempt to recall what is understood from her expounding and explaining. Then, her tablemates begin to change the numbers in this horse problem, testing a formula. Katrina becomes an *expert*. Not only does she take up more elapsed time with a correct solution and answer; not only are her words rehearsed with feedback; not only is she applying her solution as a *teacher guide* for different numbers; others view her as possessing expertise.

About two minutes after this transcript starts, Katrina begins to reflect aloud with her tablemates on the previous conversation. She is exaggerating her head movements and hitting the table with her hand, saying, "No, he does", like in a comedy. She recalls the episode of their disagreement. Everyone at the table laughs. It seems the time for exploring the arithmetic of the horse problem is over. The time for developing different solutions is equally over. These moments are about closure to the offered exploratory task of solving the horse problem.

The general noise of the class is increasing, similar to Katrina's table. Three minutes after this transcript starts, Katrina's table is beginning to talk about other things: off-task.

An introduction to "being the teacher"

The teacher begins to shift the students' focus of attention from the circular desks to her focal circle. She starts talking while many others are still talking.

> OK let's have your attention please? [To a table near the front of the classroom, replying to a question] What is the right answer? Tell me. [Addressing the class again] OK. So. This is such an interesting problem, isn't it, because it is not really hard. I have done this with grade 3s. And I will show you how in a minute.
>
> But, we had some really good discussions and after you were explaining or justifying – some of you very passionately. Which is good and the kids do that too. You know it is good to be passionate about something. We also have to learn though that with the other point of view that sometimes we have to work on this with kids because they get so set they cannot hear the other way.
>
> So, what I would like to ask you now is if anyone would like to come up and share their solution. And just record what their thinking was. How they arrived at their particular solution. And you get to use the nice overhead pen here and write and be the teacher. So, who would like to come up? Oh, she was very passionate. OK, and is that Katrina? OK, Katrina is going to come up and share with us her solution.

Before Katrina begins, she and Elizabeth momentarily talk quietly. Katrina requests the Horse Problem acetate, then begins, "Well, this is how I did it". The ink in the overhead pen does not flow with initial pen strokes. Elizabeth seems to interrupt a false start. "This is good practice, isn't it." Katrina answers Elizabeth and then, looking at the overhead and the horse problem acetate, tells the class her solution. As the ink flows she adds up the two numbers the man spends.

50 +70 120

After Katrina writes this expression, the teacher voices a short comment. "OK. That is right."

"Is that right?" (Katrina)

Elizabeth affirms her.

Then Katrina looks towards the rest of class. "I thought I made a mistake." She is covering her mouth with her hand, similar to actors when exaggerating a *faux pas*. Elizabeth again affirms her. "No, that is OK Katrina. You are doing great."

Then, Katrina adds the two numbers the man receives.

50	60
+70	+80
120	140

As Katrina writes each number, she expounds its significance by paraphrasing the problem. Katrina also turns away from the class to look at the original horse problem acetate sheet four times – once per number.

"So, to see if he gets anything or lost, you just subtract the smaller number from the larger number and you see he is positive 20. So he gains $20." She circles her answer.

50	60
+70	+80
120	140
	140
	−120
	+20

After Katrina finishes, she reconnects the pen with its cap, places both down on the overhead trolley, and returns to her table. Katrina attempts to leave the overhead space as she found it. Upon her arrival, a tablemate says, "You are a good teacher Katrina". Elizabeth seems to concur: "… very clear. Thanks Katrina. That was a very clear explanation of your thinking."

Summarizing five other students' "teaching"

The teacher invited five more participants to her focal circle to present their solutions. The following chart illustrates the mathematics each person wrote while explaining his or her solution. During the original presentations, a verbal description of the individual's mathematics includes the significance of each number. In each instance, the solution was written as the horse problem was re-read or paraphrased. Sometimes, the student added logic and understanding in between the sentences or ordered ideas of the problem. Finally, the recorded temporal order for the calculations in examples two, three, four, and five is from left to right and from top to bottom.

Presenter	His or her mathematics
Two (First Attempt)	+10 –10 0 $30
Two (Second Attempt)	50 +10 50 –20 –10 50 +30 +20
Three	Bought –$50 Sold +$60 +10 Bought –$70 –60 Sold +$80 +20
Four	$50 sold $60 – $10 $60 bought $70 – $10 sold $80 – $10
Five	Deal 1: -50 + 60 = +10 Deal 2: -70 + 80 = +10 +20
Six	$60 $50 $10 $60 $80 $10 $70 $70 $10

Summarizing others' reporting by their written mathematics, when they were instructed to "jot", serves to illustrate a comparison of Katrina to others. At Katrina's table, Katrina became an expert by telling, explaining, answering questions, clarifying, and perhaps simplifying. These actions and localized environment seem significant.

Also possible categories for these solutions exist. Some solutions point to the writing as summarizing a progress of thinking ("student jotting") more than summarizing a solution. If authors refer to their writing to develop an understanding of the problem, then their writing and associated assumptions, not their thinking (such as metaphors), became a point of focus. This seems to manufacture a persistence of an author's misunderstanding within her or his writing.

The presence of the teacher

Instead of using the words 'reporting back' to label the student activity, Elizabeth uses "teaching and "being the teacher". Being the teacher might imply that pre-service teachers are to act as Elizabeth does, with her professional intentions. If this is true, we might receive insight from Elizabeth's actions; what students might learn. Various differences, comparing the teacher's actions with those of the students-as-teachers, also provide an illustration of Elizabeth not actually giving up the role of *teacher*.

Elizabeth never presents a mathematical solution, explained with her authority as teacher. (She provides the correct numerical answer during the closure of a second horse problem activity.) Not unreasonably, only Elizabeth talks about mathematics education issues and offers helpful hints about how to solve this problem or classroom teaching. So these might be seen as teaching functions to do with this particular class (with its double agenda of teaching future elementary teachers). But the students-as-teachers did not take on any of the meta-commenting functions with respect to their own solutions they were presenting. They did not check with their audience whether they had been understood. They checked with Elizabeth who was standing nearby. These instances occurred when others were standing in the focal circle waiting to present or sometimes when they finished presenting to the class. If a presenter reports about his or her experiences or personal opinions, Elizabeth interrupts. All of these features are characteristic of students reporting back with the teacher present (Pimm, 1992) and reflect the same confusion with regard to where they are supposedly attending, to whom they are supposed to

be speaking and with what authority they are supposed speaking. It is an artificial setting with its own challenges.

During presentations, Elizabeth spoke loudly (so all could hear) asking questions and interjecting comments. Everyone else attends silently without movement, facing the presenter – like a theatrical audience and unlike what they use to do when Elizabeth presents. She is the only non-presenter who walks into the focal circle regularly before or after presenters, reflecting her privileged teacher status. She is also the only one asking the checking question: "Does that make sense to you?" Likewise, she is the only person inviting questions for presenters.

In between presenters, Elizabeth orders or straightens presentation materials, changes the acetate sheet for presenters to write on after it is full, or, at other times, simply touches some objects. These actions maintain her authority in this space.

A subsequent conversation with Katrina and her tablemates

I was quite struck at the time by this period. I wanted to explore further what sense the students at Katrina's table had made of her presentation, as well as Katrina herself. The first collection of quotes below provides some further insight into the comment "You are a good teacher Katrina", given by Katrina's tablemates.

> I think when before Katrina went up, she explained how she got the problem to us so we already knew. So, when she went up we were like, oh that is good teaching because we got it right away because she had already explained it to us. So it was the second time Katrina had explained it to us.
>
> But everyone was like, oh, when she did it. Like, it was simple.
>
> And she knew what she was talking about right away. And, like, it was right there, right there, right there.
>
> As in she wrote it nice and vertically where as other people wrote 10 here and plus or negative.
>
> I didn't think about it as plus and negative.
>
> I think, visually, this is just obvious. Oh, yah, 120, 140, minus.
>
> There is also he lost, he gained, and the net.
>
> And the words you used. He lost this much. He gained this much. Then subtract them. The words you used when you were explaining it to the class.

The qualities these future teachers are referring to are the clarity ("nice and vertically") and immediacy of the presentation (evidenced by her command of her explanation as well as them getting it right away): her "teacher" actions named as "explanation". They attribute success to the vocabulary she uses and the visual presentation of the calculation. The opening comment is particularly interesting and illustrative of polished mathematics as pedagogy. It reflects an awareness of being in a privileged position (relative to others, as well as Katrina having been able to

rehearse in the small group). Yet, the fact of "getting it right away" seems of paramount significance in this judgement. Finally, Katrina herself said:

> I think in university we look past the professor and ask, what am I supposed to be getting. As younger elementary students, we needed to be engaged. We [as university students] are just on auto-pilot.

There is an irony here, which has to do with part of Elizabeth's (and other methods instructors') repeated claim that it is precisely their actions as teachers that they wish to be in focus. And yet this tension concerning who the student actually *is* in such classes is clearly present.

Individually, after the table group discussion had ended, Katrina commented to me:

> It's ironic because when I am in the classroom I am like, look kids, look at me. But up here I am like, whatever. I wouldn't feel free in being, kind of, a circus performer here, you know. Like, isn't this great, 120, 140. I just sort of say what I have to say, you know, like I do not feel like I have to not be a show, but be obvious and enthusiastic about it. It is just, well, this is what you want to know; there you go. (Katrina)
>
> Elizabeth does exactly what you just described. (Mark)
>
> That is true. (Katrina)
>
> So, I am wondering, why the difference. Why are you uncomfortable and Elizabeth is comfortable? But, you are also saying you are very comfortable given your own class. (Mark)
>
> Yah, umm, I guess I just didn't want to seem almost like a show off, you know. Because I was pretty excited I had an answer but I didn't want to like, this is what it is. I just wanted to, plop. This is what I got. I just didn't want to be, I guess, too excited about it because then, you know, it would seem like I was a show off, you know. I am sort of looking for a low profile. (Katrina)
>
> Yah, like a student. (Mark)

> Yah, Yah. And, I mean that is something that we – I talk about with people in my other classes. That it is kind of hard for us to switch between teacher and student mode, you know. Because, I do not know, we are just, one moment we are students and then the next moment we are all sort of competing for, you know, the teacher role. And, yah but I guess I definitely play the student role here. Because I was just, you know. I even asked Elizabeth like, "Did I do something wrong; did I do this right". (Katrina)
>
> I noticed that too. But, you were not the only one who checked with Elizabeth. There were a few other students who checked with her. (Mark)
>
> I guess it is all about confidence. I didn't want to stick my neck out. I wanted the feedback all the way through. (Katrina)
>
> We decided to send emails to finish our interview.

In the above, Katrina talks about the role confusion in her education classes, where there is this double position sometimes available, of both student and teacher. What seems important to me is that such switching is not simply something that occurs. It is an explicit and significant part of the framing of such classes, where discussion of what is involved in being a teacher takes place. And yet the very clear social norms at work that collide at times in what it means to *be a teacher* and to *be a student* might suggest some problems here.

An email from Katrina

Hello Mark,

I'm glad we are doing it this way, since having a bit of time after our conversation gave me time to think about things and clarify them in my mind. [You write:]

>What do you mean by a circus performer? There are other types of
>entertainment like movie actors, theater, and street performers. What is it
>about a circus performer that you identify with when you reflect on
>teaching?

I suppose I chose the parallel of "circus performer" because it best exemplifies the high energy level I have when I teach Elementary school students. I keep my energy high because I feel that being excited about the material keeps the kids engaged and interested. Perhaps, subconsciously, I feel that I have to be as interesting as the highly visual, audio videogames that young kids play with.

>What do I hear in your other phrases about a certain hidden student-
>curriculum of behavior? What do you mean by "show-off" and "keep a low
>profile" with excitement when presenting as a student? What are your
>impressions of your friends' meanings with this? What do you remember
>your friends saying about this conversation?

I definitely think there is a hidden curriculum when it comes to student behavior. In Elementary and Junior High especially, I felt conditioned by my peers to not be a "show-off." I was a fairly good student, however, so soon I learned that if it was required of me to demonstrate something to the class, that I should do so without seeming arrogant about knowing. I unconsciously switched into this low-profile mode when I was teaching the [mathematics methods] class – partly, I suppose, because I was quite confident that I was right. I suppose, ironically, that the more confidence I have in what I am teaching to my peers, the more likely I am to show an under confident exterior.

I suppose what I'm speaking of has much to do with the social survival of a student, and those who want to survive academically as well as socially learn to live by it. I know I have learned to behave differently in different teaching/learning situations. For example, when I was doing my [earlier field experience], if I was alone in the room or my mentor teacher was obviously absorbed in a task unrelated to my instruction, I would feel free to teach as I liked and acted confident and in control. If she (my mentor teacher) was participating in the class in some way, I found myself often checking with

her – visually or verbally – to ensure I was teaching in the way she approved of. I suppose that by doing this, I was simultaneously a student and teacher.

I am also reminded of another time I filled both the student and teacher role. It was in my German 100 class in my second year. I had the best average in that class, and the teacher once asked me to teach a concept to the class. Being an Ed student as well, I naturally thought about how I would teach it beforehand and formed an informal, short lesson plan. When I went to teach it, I found my teacher constantly interjecting – almost like she was reluctant to let me actually teach. Although I understood that it was her classroom and that she had ultimate control, I still felt like she wasn't allowing me to do what she has asked. Due to her interjections, my lesson plan became choppy and I had to basically toss it out the window and improvise according to her prompts – leaving me feeling a little embarrassed. I suppose this is why I was so submissive to Elizabeth in [mathematics methods] the other day – I have learned that in such a situation it is best to let the dominant teacher direct things, and to just follow their lead. I realize that she and my German teacher alike had certain goals they wished to reach with the class, and so interjected, but nonetheless I have learned that my own plan is pointless in a way, unless they specify that they will not interrupt in any way.

This all relates to the hidden curriculum of pre-service University teachers. We learn to seamlessly switch gears from being a teacher, to being a student, to being an amalgamation of both.

Perhaps my views in this regard are partly due to the fact that I have not really mastered the art of co-teaching. In my teaching experience, even when I have taught with another it was always "one teacher, one support." I was paired with another student teacher during my [first field experience], and found that even when the mentor teacher was out of the room and it was just the other student teacher and I, one of us would always be running the show and the other would provide support where they could according to the other's wishes.

Well, I think I've pretty much covered everything. If there's anything else you're wondering about, or if I missed something, feel free to send me an email.

Katrina

Most of this exchange seems pretty self-explanatory. I wish to highlight one idea. Katrina assumes Elizabeth's framing that Katrina was "teaching the class" when standing in the focal circle reporting back on her mathematical work.

An interview with Elizabeth

Through a series of quotations, this concluding section attempts to illustrate what Elizabeth attends to as a teacher during this episode of public reporting back, as well as some of the beliefs she was trying to instill through this experience.

(a) They were really listening to each other when the kids were coming up.

(b) But that takes a while for them, you know, for a group of people to feel comfortable enough doing that. And I know some of them think they want to be careful asking another student a question because they know what it is like and they do not want to, you know, put them on the spot.

(c) But they will get there. They will get there. And lots of encouragement and it's OK and it's all right even if you get muddled a little bit, it is all right, you know, it will happen to any of us. We will be fine.

(d) It is OK to ask a question but it is also OK if you cannot answer it right now.

(e) [Presenter Number Two, first attempt] had really come right out on a limb when she went up there and did that [reporting back] and, you know, wasn't as clear as she thought she was. And yet, she was still OK with it. And, umm, but I want to make sure that when students take that risk that if something happens and they start feel uncomfortable that I make sure that I reassure them because that is also reassuring the rest of the class that, you know, I am not going to let them look stupid or we are not going to judge them or those kinds of things.

(f) All of those things are what is essential for them to realize to feel safe in my class. And so they also realize they have to create that environment then if they want to get the kids to contribute in that way. If they want the kids to start taking over that teacher roll and if they want the kids to feel good about contributing their views even if they are not 100% sure whether or not they are right. Then, there are certain ways they, as the teacher, have to act. And that is where that student-teacher thing comes all the time. You can't divide it up. What I do as a teacher influences them as students. But it also influences them as teachers. Because I am trying to show them that and build an environment that it is safe as a student but also where they see what is essential for them to do as a teacher.

(g) *Mark: What message do you think is sent to the rest of the class? Like if the branch would have fallen and [Presenter Number Two, first attempt]*

> *would have stood there unable. She had the wrong answer and her reasoning was flawed. Most of the people knew that in the class, yet the branch didn't break. What would have happened if it did? What would have been the message?*
>
> For some of them, for some of the students, they would be immune to that. I mean, it wouldn't affect them personally all that much. But I would say at least half of them it would affect. And it has been my experience that how it would affect them would be that, 'I am sure glad I am not her and I am not going up there unless I am really really sure that I am going to be right because that was really uncomfortable for her.'

Again, a number of these comments are informative and insightful, both in terms of what atmosphere Elizabeth was attempting to create, but also in terms of more specific things to do with reporting back. She was trying to scaffold an experience of speaking about their own mathematical reasoning in public to the class. Her overall concern was one of care, of attempting to ensure that the experience is not an unpleasant or unnerving one. Long (2008) has written a doctoral dissertation on the challenge for an elementary teacher of caring for students while caring for mathematical ideas at the same time. I think in the episode referred to in the final exchange, there is an instance of the same challenge at the pre-service elementary level.

> In (f), when Elizabeth says:
>
> And that is where that student-teacher thing comes all the time. You can't divide it up. What I do as a teacher influences them as students. But it also influences them as teachers. Because I am trying to show them that and build an environment that it is safe as a student but also where they see what is essential for them to do as a teacher.

She is claiming that her students are watching her *as a teacher* and are learning from *how she teaches.* Yet, it is worth recalling that Katrina said, "I think in university we look past the professor and ask, what am I supposed to be getting". One complexity

in methods classes such as this one is that some of what a student is "supposed to be getting" requires direct and deliberate attention to the teacher, and also perhaps that this student can negotiate successfully the peer pressures of being a member of the class she or he is supposedly *teaching*.

Chapter 7: A Classroom Argument

This chapter explores a rare occurrence, once during the semester; a loud argument. In the last chapter, I indicated that disagreement among students at their tables certainly happened. Three features of this interaction are different. It was more public, it was loud and sustained, and, to my ears, serious. (Serious in the sense that agreement or maintaining social relations or other desires did not soften actions.)

I feel fortunate to have such a rare occurrence. Compared to illustrating daily routines and norms of an averaged period, this transcript reveals a major disruption in the daily rhythms. It illustrates a boundary example about how language, verbal and non-verbal (body), can direct, perhaps control, a class. Because it was so rare, and even then lasted 30 seconds, it also illustrates the power of the routines used to establish and re-establish daily rhythms. And these rhythms were the same even though it was mathematics education issues and not mathematical ones that were under discussion.

Introducing the transcript

This episode occurred half an hour into the second class period of the fourth week.[17] Students had finished grading examples of elementary-student student work by using a rubric (a class handout). The transcript begins with a minor transition: a summary about this class's work and the resulting class discussion. Since it begins a couple of minutes before the actual argument, it allows some perspective on how the argument began as well as illustrating some routines.

Working through the written transcript necessitates vocal descriptions used as cues about my attribution of intentions. Because I wish to attend very closely to this thirty-second dialogue, I pay careful attention to various linguistic items. The following is a glossary of notations used in this transcript to reflect aspects of the spoken features.

- , means a short pause, read like a comma
- . means a slightly longer pause, read like a period
- .. means a 1–2-second pause
- … means about a 2-second pause
- …. means a greater than 3–second pause
- (?) means a word or sentence where the voice is raised indicating a question
- (/) means a raised voice tone different from a typical question
- (\) means a decrease other than a typical statement or understood grammatical period
- (-) means a specific word is pronounced slower than other surrounding words due to letters being articulated for longer;
- (--) means a specific word is pronounced appreciably slower than (-);
- (_) means a sentence is said with longer pauses between words and letters of words are articulated longer;
- (-_) means a sentence requires more time than (_);
- (^) means a sentence is noticeably faster relative to the sentences before and after;
- bolding signals when one person talks at the same time as another (person identified as one overlapping is second in the transcript);
- square brackets indicate my words added to the transcript to facilitate the reading.

Turn #	Approximate class time and speaker	Words heard on the audio-tape
1	32:31 Elizabeth	Ok(-). So, isn't that interesting(_).. We more or less agreed on what that [the rubric] was telling us but when we go and score it.. Huh.. Why do you say that. It's that putting that evaluation. It's that making(-) judgments(-). The assessing is fine. We are fine with the assessing. Right(?). We can see what it told us. But when we went and evaluate and put that judgment on, then we run into problems.. Huh(\).. OK(\). Yes.
2	32:58 Pat	I-I was just wondering. Like. A lot of people are saying. Well. Saying. Saying. How are we to know what this kid is thinking that when we are supposed to be grading what is actually on the paper? You can't assume that. Well, you think it has to do with it. But, you can't even use what they had said before. Yo**u have to use what is on the paper.**
3	33:07 Elizabeth	**Ok…. Excellent(/).. OK(\). Excellent Point(_).. Excellent Point(_)**.. And that is what is so hard(-), as the person who is doing the assessing… and evaluating(-). You have to be careful, that all(-) you are judging is what is given to you.. Remember that when I give you back your assignments(^). I can only judge what you gave me.. I can think to myself, oh I think they really understood, but I-I don't know. I can only base it on what you have given me.. So based on what is there.. Why would you score it a 3.
4	33:50 Susan	Well.. Me personally.. I-I think we might be a little different.. But, Umm.. The rubric doesn't say anything about a correct number.
5	34:00 Elizabeth	**Uhh**…. OK, so there is nothing about a **correct answer** (/)
6	34:01 Susan	**No**, I mean, yah, the answer's wrong **but**
7	34:05 Elizabeth	**But**. But, there is an answer.
8	Susan	Yes.
9	Elizabeth	OK.
10	34:05 Susan	So in the number sentence it is complete.
11	Elizabeth	OK.
12	Susan	Uhh, the drawing.. Minus the whole. I don't even talk about
13	Elizabeth	Yah.
14	Susan	But there is one draw from there … who knows, right(?)
15	Elizabeth	Yup.

16	Susan	And then the story, states, ongoes. So, to me(-).. I mean.. I'm(-) thinking a 3. I(-) would have been asking for a correct answer.
17	Elizabeth	OK.
18	Susan	I would be asking for a few things and he she did that.
19	34:30 Elizabeth	OK.
20	34:33 Pat	Did she count that first assignment, did she count that as a 2(?)
21	34:36 Different Woman	No. I counted it as a 1.
22	34:38 Susan	I counted it as 2.
23	34:40 Pat	Ok.. You gave, you gave, it 2-3 as well.
24	Different woman	I gave it a 3.
25	34:47 Pat	You only counted the first one as 1(-)(?)
26	34:50 – 34:56 Susan	I counted it as 2(-)(/). **[Laughter for time indicated]**
27	34:52 – 34:55 Elizabeth	**Ok. But then. OK.. OK. But. But point is. All right. So. [Susan and Pat are talking at the same time so words are inaudible.]**
28	34:56 Pat	**The**-the picture clearly and accurately illustrates.. Accurately illustrates(-).. [Reading from the rubric provided.]
29	35:00 Susan	Oh. So now they have to be perfect artists(-)(/).
30	35:02 Pat	**Ye**
31	35:02 Susan	**Oh come on**(/).

32	35:02-35:26	[Elizabeth, Susan, and Pat are all talking with quick turns. Other students also begin talking. Below is a more approximate text of what is heard.] Elizabeth: OK. Pat: The number sentence Elizabeth: Didn't quite hear you. Ok. Pat: The number sentence, one [inaudible] Elizabeth: Yup. Pat: So if it was **accurately depicted** Susan: **So honestly depicted**. Right. Elizabeth: Yup. Susan: It is. Elizabeth: Yup. Susan: You probably forget that [inaudible] Pat: What happens when it is you. Unknown: I would rather give [him/her] a 3 than a 1.
33	35:24 Elizabeth	[Many students are talking when Elizabeth is speaking, thus the bolding.] **You.. You.. You see what happens.. When things.. This is so good, you know why(?).. Because. You see what happens when you have to put a judgment on it(?). What happens..** [Muffled answer and many people laugh.] **It goes from being very quiet to being a little heated and hot here(\).. Becau. Because what happens, what happens.. What happens(?)..**
34	35:44	Muffled answer less than half a second but only one person is talking.
35	35:44 Elizabeth	**It's emotion. You betch-yah.** It's emotion. It comes in because, we are having to make judgments about.. I don't care how hard you are being trying to be objective(--)(-_)… Subjectivity comes in all of the time when we have to make judgments about what something someone did(-_).. You, it. And don't lie to yourself.. You-you are going to be subjective. The only thing you can do is be as clear as possible and as consistent(-)(/) as possible. So if Susan is marking that as a 3 then she has to use that same interpretation and criteria for all of them.. Ok(?).. Now.

(Additional details that aid in reading the transcript.)

Turn Number	Additions to transcript
3	3 is full marks on the rubric provided to students
4	Susan was previously talking about her interpretation of the rubric, her interpretation of the elementary student's work, her assigning a grade, and what numerical value it is before the transcript starts. In turn 4 she is adding to her previous, before transcript, turn.
27	While Elizabeth is talking both Susan's and Pat's vocal volume is markedly higher. Also, they are speaking at the same time. There is a moment where Elizabeth and Pat are not talking; at about the middle of the middle of the six seconds, where Susan is able to say, "that is not a one". Afterwards, Pat is responding with a repeated "ok" while Susan, again, laughs and Elizabeth starts talking.
27	Elizabeth's vocal volume is constant during this exchange. Furthermore, it seems to be less than other earlier times in the transcript.
28	Pat's vocalization during this turn does not seem to change in vocal descriptions, except for the second accurately illustrates, which is noticeably different.
29	Susan's vocal quality seems more stressed than when previously talking with Elizabeth.
32	I remember Elizabeth's use of proximity during this turn. She stands just in front of the line of sight of these two students, turning her head appropriately as each person speaks. Near the end of this turn, she stands in the line of sight, and then looks in my direction. Both students are talking 'around' Elizabeth, directly to each other. This marks a change in who Elizabeth addresses next.
33	While Elizabeth is talking other students are also talking. The first question Elizabeth asks with some wait time is inappropriately answered, and many people laugh. Elizabeth does not address the comment. She continues and the second question she asks with wait time and is answered as "emotion". Immediately afterward the student's answer Elizabeth chooses no pause. As the student finishes pronouncing the word Elizabeth is again speaking.

35	The second sentence marked with (-_) notably takes more time than the first sentence and more time than other sentences with (-_). At least one student is talking while Elizabeth is talking during this time. However, it is noticeably quieter than the exchanges turns before. After this turn ends only one person can be heard talking.
36	A student asks a question that could change the topic of conversation and Elizabeth allows this.

Some initial observations about the transcript

For much of this transcript (turns 3–19), Elizabeth provides every second response. When compared with the conventional Initiation – Response – Feedback structure of much teacher-led classroom discourse (discussed in Pimm, 1987), this seems similar. Students voice their contributions and the teacher provides agreement (as 'feedback') without reinitiating.

When Elizabeth switches from "you" to "I" (turn 3), she also switches from the elementary student work as context (with students acting as teacher-graders) to her as their teacher, which positions them back as students in *her* class. The "you" comments in this turn seem to point towards teacher-graders (and perhaps other turns if this role persists).

Rewriting the transcript

The transcript begins when Elizabeth summarizes what she believes students have done and where they need to attend. "We more or less agreed on what that was telling us" seems based on a public response of raised hands. From a rubric out of three, different students score a piece of elementary-student work: the class "more or less" agrees. This far, the class has experience with this rubric, grading, and noticing each others' assessment values. In all instances, students discuss a meaning for and an application of the rubric with an opportunity to change their numerical values. Many students do change their assigned grade after class discussions.

After Elizabeth transitions with a "Huh", Pat makes a slow start to asking his question. His question does not repeat words she uses in the previous turn. His

question points towards routine phrases she has used since the beginning of the course: attention to thinking. It points to his misunderstanding about her words concerning how attention to thinking can occur through submitted work. Pat also seems to anticipate an answer when he includes repeated words in his slow lead-up.

Pat challenges Elizabeth. After he states his question, his pronouns change. He seems to address her directly. Then, he answers his own question.

Elizabeth seems to possess urgency for voicing her response and, perhaps, ending Pat's turn. Also, the words "excellent" and "excellent point" seem to direct students to something of value. It is certainly evaluative 'Feedback'. On the surface, it seems her response repeats many of the words and emphasis of turn 1 without addressing his question. She repeats his answer. Her pronouns change and she points to Pat and assessing only what is on the paper. Then, she describes a personal and immediate situation about narratives teachers create about students' writing and how these sometimes lack the developed thought patterns illustrated in other ways. This situation seems to further support Pat's challenge as valid, not to be ignored. By the time Elizabeth talks about grading, with "a 3", her response seems quick.

Susan answers Elizabeth with a continuation of a dialogue that started before the transcript begins. Pat identifies a partial ending and asks another question. Students do not normally address students outside their table. It is routine for a student to talk about another, through Elizabeth. Elizabeth encourages this.[18] However, Pat's question's unique specificity ("she") encourages Elizabeth not to answer. Thus, a rare cross-table student-to-student dialogue begins.

Pat begins to layer intentions without saying words directly. "You only counted the first one as 1," (turn 25) layers two ideas. First, counting the first one as 1 wonders about reasons. By including "only" and taking more time to pronounce "1", Pat conveys a mistake, perhaps a wrong.

Arguing about a meaning of a rubric is normal when vocal volume and tone are communicating respect and value. Here, the vocal volume and tone seem to communicate something else. Elizabeth attempts to re-establish her routine of students talking through her about ideas.

During turns 26 and 27 Elizabeth uses other conjunctions to cue a checking-in effect. She also lowers her vocal volume, seemingly balancing her use of proximity (moving closer to the two arguing) and her responsibility of authority.[19] She is *simply* not successful. Susan and Pat do not provide power for her success. Both Pat and Susan seem more personally-than-community motivated, especially given Pat's *ad hominem* (turn 32).

Thirty seconds after this unique classroom episode begins, Elizabeth forcibly ends it. Elizabeth has tried standing in front of the line of sight between Susan and Pat; her head turning, attending to each person speaking. Now, Elizabeth steps on this sight line and looks in my direction. Then, addressing the class, ignoring Susan and Pat, the words of turn 33 begin.

"You" is repeated six times before Elizabeth finishes a sentence. "What happens" is also repeated six times during the turn. This individualized meta-perspective seems to increase a perceived distance from the frustration of the argument. While Elizabeth forces a choice of attention, she also cues a familiar

pattern of not continuing after a question. Turn 35 expands on the "emotion" answer. However, before Elizabeth refers to a rubric there is a question about whose emotion, where it started, and the difference between subjective and objective.

Pronouns: an issue of perspective

The following table contains a list of pronouns from each turn. When phrases or words are used like a pronoun, I have assumed the pronoun and illustrated it within square brackets. 'Implied' means the particular turn does not contain a pronoun; the speaker seems to imply perspective in others ways (such as gaze direction). Pronouns cue perspective by guiding or directing listeners to a perspective the speaker intends his or her thoughts to be perceived from. Pronouns seem to generate a perceptive, top-down, process to initiate a certain way of interpreting the text.

Using a word processing tool for counting words, the following provides an initial impression about pronoun usage. Susan and "a Different Woman" use about the same percentage of pronouns as they do percentage of total words. Elizabeth's percentage of pronouns is higher and Pat's is lower. Elizabeth is also the only person to say each pronoun listed at least once. Relative to other speakers: Elizabeth uses "we" and "us" more; Elizabeth and Pat use "you" more; Susan and "a Different Woman" use "I" more. Elizabeth and Pat also tend to mix perspectives during a turn more than Susan and "a different woman".

Turn ID Number	Speaker	Pronouns from Each Turn
1	Elizabeth	…we…us…we…you…we…we…us…we…we
2	Pat	I…[them]…we…[him/her]…we…you…they…
3	Elizabeth	[he/she]…you…you…you…I…you…your…I…you…me…I …myself…I…they…I…I…you…you…you
4	Susan	Me…I…we…
5	Elizabeth	Implied
6	Susan	I
7	Elizabeth	Implied
8	Susan	Implied
9	Elizabeth	Implied
10	Susan	Implied
11	Elizabeth	Implied
12	Susan	I
13	Elizabeth	Implied
14	Susan	[Anyone]
15	Elizabeth	Implied
16	Susan	Me…I…I…I
17	Elizabeth	Implied
18	Susan	I…he she
19	Elizabeth	Implied
20	Pat	She…she
21	Different Woman	I
22	Susan	I
23	Pat	You…you…
24	Different Woman	I
25	Pat	You
26	Susan	I
27	Elizabeth	Implied
28	Pat	Implied
29	Susan	They
30	Pat	Implied
31	Susan	Implied
32		ARGUMENT
33	Elizabeth	You…you…you…you…you…you…
34		Unheard and muffled
35	Elizabeth	You…we…I…you…we…[he/she]…you…yourself…you… you…[she]…she…them…

A subsequent meeting with students

I arranged to talk further with some of the students involved in this argument. The following transcripts use a minimum apparatus of punctuation. Periods and commas refer to pauses, as before. Semi-colons refer to a pause that seemed in between a period and a comma. Question marks refer to a sentence that sounded to me like a question. An ellipsis refers to my ending a turn before the speaker.

Pat sits at a table with Benjamin. At this time during the semester, Pat regularly talked with Benjamin and they often left the classroom together. I did not realize Benjamin would have as much to say about the argument as Pat; enough to suggest Benjamin's active participation.

In Pat's opening words, he seems to consider having an argument and having a conversation as the same. "I had the argument. I had the conversation." Pat describes his "conversation" as "heated" with "emotions flying".

Benjamin offers the next comment.

> I think when you guys were almost. Everyone knew you had the floor. So, I think everyone was like, OK, well, we are not going to talk until these two figure it out. No one was going to jump in right.

"…were almost" is unmistakable. There is something, not "conversation", that Benjamin would have said next. My suspicion is it would have been *arguing* or *fighting*. Benjamin's other phrases also point to something. For example, he talks about "everyone […] not going to talk", which ignores the noise of the classroom. (Reading only Benjamin's comment one could presume "they" is the joint referent of the 'you' and not just Pat. However, "you" seems to refer to others. See below.) They "had the floor". But they had the floor in a way different from the way that the

individual presenters had the floor who were reporting back (in a setting and space sanctioned by their teacher) described in the previous chapter.

Many students were talking in low voices, and some comments were about Susan and Pat, but none of this speech was *public*. Another example is, "no one was going to jump in". It is Elizabeth (as the teacher) who "jumps in". Finally, Benjamin consistently uses plural pronouns indicating two arguers; except with, "Everyone knew you had the floor".

> I was egging him on. (Benjamin)
>
> Yes, he was kicking me in the shins. So after a couple of the. After a couple of the back and forths, I thought I better just let it go and you sort of just want to. You don't want it out of control. It's not a. You said your two bits and that's it. You could still be there going back and forth and back and forth. Right? Like. (Pat)
>
> It is out of respect, eh. (Benjamin)
>
> Yah, all you say what you have to say and leave it at that. You can sit and argue your point but you don't have to. I don't know, like, you don't have to prove your point of view to the other students on and on and on for hours. (Pat)

During the actual argument, Pat and Susan seem to antagonize each other. Here, Pat's switch in pronouns, from "I" to "you", seems to question who he is talking about; Benjamin, me, his "conversation", or someone else. Benjamin seems to answer Pat's rhetorical question, "Right?", and attempt to clarify what he is talking about. However, something is amiss with "<u>your</u> two bits", "<u>your</u> point of view", "<u>you</u> don't want it out of control", and "respect". After I agree with "leave it at that", Pat continues. "Yah, and I just wanted it to end there. Move on to the next one and argue that about that one. Right? There, there is more to discuss after that."

I ask Pat about what he thought of Elizabeth interrupting him and Susan.

> Yah, like I said, I just wanted to get it to move on too. And it is good that she jumped in to get more control. Not that she lost control. But, just to get, OK, let's move on, we have more to stuff to do.

For Pat, who forces everyone to attend to his ideas, to say Elizabeth must do more to "control" him seems to refer to more than his "conversation".

Kelly is another student who is listening to this meeting. Attempting to include her, I recall an episode during this period at Kelly's table about a misunderstanding, how it was clarified, and how it did not progress into having an argument. No one interrupted or disrupted their conversation. Pat and Benjamin continue about the original argument in quick turns.

> I don't think it was a misunderstanding. I think it was a, her was, she was on this side. I was on my side. We were not given. (Pat)
>
> No one was giving any ground. You were just sitting there and (Benjamin)
>
> This is why I did this and this is why I did this and OK." (Pat)

Both Pat and Benjamin seem to repeat the boundaries about two different tables and remaining in their places. However, Pat never uses the pronoun "I" during his "conversation" with Susan. Here, in two short turns, he uses "I" five times and implies it once with "we". Again, something seems amiss. Perhaps, this is why Benjamin's next turn seems to blame "the question", a different focus.

> I think the question too was very opinionated. That, there wasn't really a right answer. If you had a point that was sufficient and could back it up then."(Benjamin)[20]
>
> If you had justification for it then it was all said and done right there. (Pat)

Yet, blame does not seem to rest on "the question".

I ask about the idea of having an argument and Elizabeth "jumping in and ending it", use the words "narrative" and "story", and not giving answers, to describe turn 35. Then I wait and listen for any connections to be made throughout the period, perhaps a routine.

> I think with the narratives she tells, I think it is a good way for everyone to get back on. I think everyone listens to personal experiences. Like when she tells something about what she has done before, what she has seen before, it gets everyone sort of back on to topic, gets them refocused. Gets them off of what they were talking about before. Everyone is interested to hear about past stories. (Pat)

Benjamin and Kelly talk about turn 35 too.

> I think it is also a respect thing too. Like, she is the teacher and we have gone through school respecting the teacher and being guided by them through the class. And, if when she kind of speaks up, we say we better settle down and get back to what she wants and is actually at hand. What she wants. (Benjamin)

> And I think when she does that we remember. It brings you back to the fact we have to respect that we are not all the same way and we have to respect each other's opinions... (Kelly)

After Kelly's turn, Pat and Kelly initiate a summary by talking about the lesson.

> That was a good class because we did talk about respect and the rubrics and everything and how emotion does get into marking and how we have to leave that out when we are marking. (Pat)

It seems Pat's words, here, ignore his words about disagreeing with Susan.

> And a lot of those come from how school has taken us. (Kelly)

It seems Pat will not be remembering anything frustrating about having an argument today. I will. Perhaps Elizabeth will too.

Kelly's agreement with Pat

This section's purpose illustrates how a student can think, talk, and direct a class, or smaller group, away from what the teacher intends (in this case what a researcher intends). This turn happens after Pat and Benjamin disagree with my suggestion of a misunderstanding, comparing Kelly's table with the argument between tables. Kelly is agreeing with Pat.

> It is one thing, yah, to have something like I misunderstand you and. Yah, I agree with you completely. I think we can get. I think we have to be careful because, our decisions, how we react and treat those kids. The messages that we send those children are far more powerful when we do not speak. You know, when we are marking those if the core of our teaching is based on, you know what, you can do this; and, I am here to help you I am here to help facilitate you instead of fill you up. Then, we must be careful how we use the rubrics. And, you know, this particular rubric, I thought of it more as this is to keep me on the straight and narrow. It is not necessarily that I need to use it as it is written. It is not necessarily that I have to use it but it helps me to stay within the rubric. And I have to know before I use that rubric what I am looking for. Am I looking for what the kid knows or am I looking for certain things in particular.

This seems like a narrative about how Kelly understands Elizabeth's lesson. As Kelly spoke, she did amaze me with how easily her words flowed. Yet, a question remains about what a person (me) does with a text like this just after it is spoken. How does a person responsible for initiating continue when someone's ideas exist outside a meta-perspective?

I focused the group away from this text. Perhaps my focusing created a boundary for Kelly since she does contribute other comments. Although, I did invite her to speak with me after our group's time ended, this did not occur. After leaving through the classroom door with my belongings I passed Kelly, Benjamin, and Pat. As a different group, they continued reflecting on this period's lesson in another

doorway's threshold. I still wonder about that boundary and what Kelly was attending to.

A meeting with Elizabeth, one hour after class

Talking with Elizabeth illustrates one idea about this sort of argument from the teacher's perspective: it happened, lessons are learned, and it must not be reflected on publicly. Following the same ideas of a researcher suggesting and waiting, Elizabeth and I come close to mentioning the argument three times during our discussion. As Elizabeth demonstrates that she will not reflect on this event with me beyond repeating a confrontation philosophy similar to the Horse Problem argument, I gain a sense of clarity about an impact of this episode.

Chapter 8: In Conclusion

The purpose of this final chapter is to highlight intentions and intentionality of this classroom and of this beginning researcher, me. One significant issue with qualitative research, especially about opening a classroom door wide enough to allow a researcher in, is "only [finding] out about the particularities and contingencies of that classroom". (Pimm, 1994, 134) Rather than solely summarise my study in this chapter, I prefer to remain true to my intention previously described, to find out more about potentialities for my own teaching from studying someone else's.

For example, Pimm (1992, 1994) wrote two pieces about the mathematics classroom phenomenon of reporting back and has pointed to some of its particularities, challenges, and confusions. To use this writing as a filter for my observations about one episode of mathematics in an elementary methods classroom ignores many intentions (such as those from other data chapters). By considering intentions and actions, this research highlights an additional feature, namely how the classroom complexity compounds when reporting back is framed by the teacher-educator as an instance of "teaching".

Researchers such as DeFranco and McGivney-Burelle (2001) describe and account for mathematics classroom intentions in different ways, pointing to at least one intention that, perhaps, we all struggle with. "Results indicate that the [students] appear to adapt a new set of beliefs regarding the teaching and learning of mathematics, yet did not draw on these beliefs to inform their teaching practices."

(681) (The conversations reported and discussed in Chapter 6 especially, concerning the tension between 'being a student' and 'assuming a teacher role', seem further illuminated by this quotation.)

DeFranco and McGivney-Burelle point to a *containment of change*. The students can change what they write without changing what they do. With mathematics and mathematics education classrooms, different types of change exist. Clarke (1988) writes indirectly about this, relating the context of psychotherapy to that of classroom mathematics teaching.

> It is only when she [the client] becomes aware of (as distinct from 'knowing') what she does, that she can begin to act otherwise; indeed, that real awareness is, in itself, change. [//] In the case of mathematics education, if the teacher knows that a pupil has a weak concept of ratio, a good teacher will not simply repeat the definition and explain it and tell it. She [the teacher] will provide material for the pupil to work on which is chosen for its relevance precisely to developing an awareness of ratio. And she will be alongside that pupil noting the points of difficulty and helping the pupil to discover the difficulties he is having so *he* can pinpoint them, unravel them, and move on. (173; *emphasis in original*)

Most noticeable to me is my notation of // illustrating a paragraph break that recalls Robert Recorde's symbol of equality. It is my clue, after I read this passage, about how Clarke seems to intend the word "change" to be read differently in both paragraphs.

The sort of change in a classroom is unique and manufactures what Mason writes of as invoking a tension (cited in Pimm, 1993b, 28)[21]; what Mary Boole (in Tahta, 1972, 11) terms as a teacher-lust; and what Hewitt (1996), quoting Mason, refers to with, "Practice tends to focus attention on precisely those aspects of a technique which have to be done without attention when the technique is mastered."

(34) Change must be something that, in a lesson, is initiated by the teacher and, with assessment, might require further initiating (if not already present in a student) that seems like the teacher continuing for the students. (Clarke writes how difficult change is and how this can be understood as frustration interpreted negatively and relative to someone else's authority.[22]) These intentions for classroom change, some previously marked in this thesis' by the words 'assist' and 'help', become mixed (some emphasized and some ignored) according to the skills of a class: teacher(s), students, and anyone else involved.

Yet it is my belief that texts, and a teacher's initiating, re-create and affect mathematical objects' existence. Consider, for example, the final data chapter about an argument. Arguments happened at other times during the semester, but none like this. Different from the other illustrated arguments, definitions about the elementary student's work and the rubric were interpreted differently between people. No meta-interpretation is available. These differences are un-reconcilable without aggressive and brief intervention with routine actions. Also, at least one comment points to the teacher's opinionated question as a cause.

Remembering that these observations are meant for this researcher to experience himself more clearly – me more clearly – allow me to focus attention towards the routine actions of polished mathematics as pedagogy. It seems Elizabeth re-manufactured classroom norms with her words, voice, vocal qualities, physical movements, questions, and allowing another person to re-direct attention. According to one student I interviewed, these methods seem like stories Elizabeth shares, perhaps during specially designated time, connecting various parts of a period.

Shifting the setting

In order to provide some evidence of my own change and learning from time spent attending closely in Elizabeth's classroom, I conclude this dissertation as I started in Chapter 1, with a short account from my own classroom. What occurs when some students act in ways successful in other places yet inappropriate in a classroom, especially with teaching and learning? Indeed, these students respond to a request for change and communicate that a teacher's interventions are necessary (like the adult student interviewed). How might a teacher plan for these?

The mathematical purpose of this episode's lesson was formulating the perimeter of shapes using variables (approaching formal notation) while developing ideas that various formulae are equivalent (preparing students for algebra), specifically with a rectangle. As a class we reviewed the words regular and irregular and students pointed to different shapes of polygons on a page with a finger. Then, using a 24 cm value, we imagined a square's perimeter (imagining – with real props – a string, meter stick, and associated operation), drew one, illustrated its side lengths, and noted more formal notations for calculating that perimeter. Here, I recorded on the whiteboard. Without reviewing these formulae and using 24 cm, again, I provided the following instructions. "What would a rectangle's perimeter look like if it measured 24 cm? Record your answers like we did the square. You all have 60 seconds individually and then share your answers with your neighbour." The following is my reconstructed recollection.

Initially, no one moved. Some people began to speak to others and I re-directed their attention to *the rules*. Frustration set in and after 20 seconds I

disrupted my interventions and expectations and invited students to share with their neighbours. Voices seemed to erupt. Students shared ideas about this task or other topics, not answers. This provided time to write "P = 24 cm" and draw three rectangles on the overhead; one like a square, one with side ratios about 2:1, and one narrow and long. As I drew, I interrupted voices (of particular students) with, "Do you see these". Asking different, selected students, "What are the side lengths here", more students focused attention on my drawings. Within 30 seconds, the students were again quiet and raising hands. The first offering recalled our work with the square. "That might look like a square, but it is a rectangle. What change would you have to make?" Raised hands selectively volunteered various answers until one student said, "7 by 5", repeating a previously voiced answer. "This is a new way of saying side lengths. You are beginning to generalize these measurements." Using the pictures, soon we illustrated a chart of five rectangle's length and width. (Students chose to limit rectangles to whole numbers.) Then I voiced a fateful question.

"Do any other rectangles exist?" More than half the class raised their hands and, too excited to wait, began to shout-out answers. My continued response of "It is already here" seemed to set frustration in, again. Students, in at least three locations, began to argue about finding other rectangles. "No! That rectangle is already there!" Others began to voice sound effects. Others began to laugh. I laughed. "What is going on? What is so funny (attempting to point out the obvious)?" Then I initiated a routine about journaling reinforced with assessment and evaluation; one that allows us all to talk, argue, and ignore what we choose to; one at a time and privately. (Perhaps I should have written earlier the students' first task while finding their

places at the beginning of the period was, "On a piece of paper illustrate something that helps you smile and keep your journal paper on your desk … we're going *in*.") "Hold up your magic pieces of paper! … Hold *them* up!" (Surprises and the magic of mathematics existed with this class. For example, by using one's mathematics one could know other ideas, like side lengths, without counting or measuring.)

"Write down what you are frustrated about and, after a few minutes, share by whispering." After a few minutes of writing and then sharing, sometimes from students to me, I asked the class, "Does anyone have any mathematics to share about this problem?" Students spoke as if talking to themselves while addressing the class. "I didn't understand [this part] and after talking with [student] I realized why 6 cm is not on the chart." Or, "The reason 11 cm is the largest number is …". One student even said, "I know there are more rectangles, I just don't know how to say it". Other ideas were shared just before the period ended and I was able to close the period with, "Believe it or not but we studied seven pages of your textbook; hand in your journals and perhaps some of you can show us how we did that next period." I finished the period with visiting and walking students to the door, smiling and laughing as I said goodbye and thanks for the mathematics.

I still do not know what I would have done had journaling not dissipated that final bit of frustration. Or what I would have done if students had begun to argue about definitions for a square, rectangle, perimeter, or other choices similar to the argument about rubrics. Or what I would have done if students stopped their mathematics after believing they did not possess the experience with perimeter as the textbook presented in multiple pages. I do know that the journals did not

illustrate, in writing, what many people voiced. And, the next period, most could not paraphrase or add information to two students' *incomplete* summaries about studying multiple pages. (A few students recalled funny bird calls.) This might demonstrate that lesson as a failure. I also know and believe, "as distinct from 'knowing'", students would change because of our experience since change was initiated; and, constant attempts to remember that period in the context of those seven pages of arithmetic and writing, practicing what would need to be automatic, would also remember that change. Summative assessment questions like, "Find the perimeter of … given …", or "Find the side length of … given …", or a combination could also assist with knowing that change. Even a different question would do the same.[23] And these only illustrate a type of change in a classroom. Also, I know this change was possible in part because of what I learnt from this study.

A client in therapy, as Clarke described, might measure success as acting differently, appropriately, efficiently, and effectively while experiencing frustration. While a mathematics exam may require these actions, for many teachers, change is measured when repetitious questions, such as the previously written or different questions, are answered correctly. Furthermore, a teacher communicates sometimes with a class (the teacher being a member) and sometimes with individuals, often at the same time. With these threads, change in a classroom is amended and defined.

'Going meta'

Through writing the different data chapters, my changing perspectives and distances relative to time and the varied 'texts' at work in a classroom may be considered practice with *meta*. This seems to communicate an approach to something I am not yet aware of. Drawing on Pimm's use of the notion of meta-commenting provides one obvious instance. Clarke (1988) also addresses *meta* by summarizing her experiences with a type of ignoring: the more a person can focus on the content and ignore various texts, not caught up with any one set of details, the more a person can address underlying processes. (Someone, for example, caught up in details of her or his rubric may ignore a possibility of other rubrics and a pedagogical issue about arguing aggressively.)

In this thesis, by illustrating differences between texts, time, exercise, influence, and wondering about different methods and actions, *containment of change* points to certain highlights in my development and progress as a teacher. Living through classrooms with increasing awareness for changing action seems a way to change mathematics teaching and learning. Perhaps this suggests a way to change mathematics education so classroom actions keep pace with other subject-discipline classrooms (at least my reading of these changes that include developments of teacher resources and professional development).

If I attempt to summarize the variety of texts teachers and students use, I will, invariably, describe one text of a classroom. I will also miss various intentions that include others' experience of these classrooms. However, if this summary

serves to problematize then I may motivate action through a paradox – a distinct way of knowing beyond knowing – and include unknown intentions by whispering.

A Narrative Coda: Some After-words

Like in the interlude, I attempt to weave some threads to assist a closing orientation of the reader in the direction I intend. In the previous interlude, I ended with a story about a hat. I will return to this shortly. For now, I address a question of authority and what comes next.

This sort of thesis produces an end product that is not a collection of words but a changed person: me. Yet, it is this thesis that may well stand in front of many more than I physically can. As a teacher researching teacher-education returns to a grade-school classroom (and continues thinking about teacher-education), I have learned to recognize that I tend towards taking care of mathematics more than of people (see Long, 2008). This seems, for now, neither positive nor negative but another text for negotiating and then ignoring; then, using changed actions to continue teaching and learning.

One example text is as follows. Teacher educators, and teachers, design a complex curriculum in every course. Two strands weave endlessly. The first is teacher-educators teach pre-service teachers (or students) to act like teachers teaching mathematics (or to act like mathematicians doing mathematics). The second is teacher-educators teach pre-service teachers (or students) to act like teachers teaching young people (or to act maturely assisting peers), especially within elementary education.[24] In teacher education, these strands seem like a weave tightly woven as topics of mathematics and topics of young learners begin to suggest actions, tasks, activities, projects, and assessment appropriate of mathematics and of

young people learning to communicate. And this presumes teacher-educators can touch both mathematics and young people. In fact, these are imagined. What teacher-educators do touch are students (pre-service teachers) and the various resources that facilitate mathematics and specifically mathematical objects meant to convey elementary mathematics. The design is complex and, at least to me, requires a sense of problem solving. I would name this as knowing about paradoxes: perhaps layered texts that seem contradictory until perceived from a different perspective and with different awareness.

These statements are made with my responsibility to continue thinking and researching, and an authority. However, there is a context about this authority. (And often I seem to speak and write outside these boundaries.) The stronger statements indicate I should negotiate it more. I believe the following story illustrates "negotiate it". During this thesis' defense, I made the following statement.

> As a researcher, I discovered through studying others I am limited to observing what I know and understand (perhaps by the intentions of this study). This provides, if these words convey well enough, a way of intuitive knowing through paradoxes (at least to me), especially when I work through others' research for ideas, understanding, and knowing.

I presume the defense chair, upon hearing this, heard whispers of other texts and, presuming she recognized a readiness, intended to assist me. She quoted Maturana (1988). " …an observer has no operational basis to make any statement or claim about objects, entities or relations as if they existed independently of what he or she does." (30) Within a conversation, this echoes both a compliment and a request: further reading. It also whispers a connection. For example, I once heard an influential and modest Professor Emeritus say (while eating lunch together), "I live

with Maturana." Maturana seems an introduction into something more (and writers like René Thom, Caleb Gattegno, and Dick Tahta).

Later, the defense chair asked me a more general and, at least to me, troubling question. "Mark, what does this [thesis] mean for teacher education? And keep in mind you are responding to people who have made teacher education their careers." I paused with this question. Answering with words like seem or perhaps, or hedging an answer, would leave a distasteful aftertaste in my mouth. I began to tell a story about buying a hat to wear for initiating different conversations, as a teacher-educator.

"This is a profound idea you are saying about teacher education. What does it mean?" I needed to bring attention to an idea about structures after hearing the chair's story about sharing circles she facilitates as a teacher educator. I told another story about drumming circles, attending to a First Nations' structure of specific seating arrangements relative to the most experienced drummer. However, I made a mistake with the order; trust about knowing seemed in question. I layered another story. I supported my son, as a toddler, drumming in such a circle (afterwards he was given the animal-hide drum stick he used). This unique and rare inclusion presents trust differently and re-defines knowing within experience and recognition from others. With knowing hopefully shifted, I could make statements about meaning.

As a teacher-educator I attempted to design conversations where students could meta-comment as teachers designing curriculum. In some senses I believe I was successful. While some students criticized, questioning usefulness, others re-framed, pointing to usefulness; as some criticized harshly, others voiced reminders

about classroom behavior. From where I sat, students meta-commented as teachers and I, as the official "teacher", did not. (At least, not until I, as the teacher, assumed a different set of actions, starting with removing my hat.) These *not* teacher actions make *protection* and *public* available to students: my intentional design. However, my inexperience made me vulnerable.

This was a stressful time. Due to a lack of better words, I required a "meta-perspective" for meta-commenting about the protected places of teaching originating from the actions of teachers. I wanted students' actions to be able to gain access to these places, and not as a planned activity. I could teach students about teaching and learning as students, and this other way seemed to address topics normally off limits; but not because students do not know about *them*. This made off-limit ideas a part of the spoken text of the classroom; and offers a reason for further reflection about *public* mathematics.

As a group in the defense room, we talked about this by sharing different stories and questioning me further about identities. I felt encouraged: my stories accepted as statements layering definition from my position and authority. Students should feel like this; and in this examination I felt like a student.

This story about "negotiate it" illustrates what I intend to come next. As I have written my thesis, writing narratives or accounts-of leads to knowing about these in ways I must revisit in a classroom. And so continues a process of telling and re-telling classroom stories.

NOTES

[1] While "a" in the title can be read broadly, Loewenberg-Ball discusses some constraints of a single course later in this article.

[2] On a much more personal note, it is here that I explore how David supported my intention of becoming in a way and with an outcome of who I intended to be, a sort of change already present.

[3] Jaworski writes, "How on earth could we let pupils know what it is they must discover, without somehow ourselves making it clear to them? However, they will certainty learn from whatever it is that they *do* discover?" (Pimm & Jaworski, 1990, 48; *emphasis in original*)

[4] The teacher (whether school-teacher, minister of religion, political leader or head of a family) has a desire to make those under him conform themselves to his ideals. Nations could not be built up, nor children preserved from ruin, if such desire did not exist and exert itself in some degree. But it has its gamut of lusts, very similar to those run down by other faculties. First the teacher wants to regulate the actions, conduct and thought of other people in a way that does no obvious harm but is quite in excess both of normal rights and practical necessity. Next, he wants to proselytise, convince, control, to arrest the spontaneous action of other minds, to an extent which ultimately defeats it own ends by making the pupils too feeble and automatic to carry on his teaching into the future with any vigour. Lastly, he acquires a sheer automatic lust for telling people "to don't", for arresting spontaneous action in others in a way that destroys their power even to learn at the time what he is trying to teach them. What is wanted is that we should pull these three series tight so as to see the parallelism, and not go on fogging ourselves with any such foolish notion as that sex-passion is a lust of the flesh and teacher-lust a thing in itself pure and good, which may be legitimately indulged in to the uttermost.

Few teachers now are so conceited as not to know that they have a great deal to learn, and that their methods need revising and improving, but the majority are seeking for improved methods of doing more of what they are already doing a great deal too much of. The improvement which they most need is to be brought under conviction, to be made to see their conduct, their aims, their whole attitude towards their pupils and their work, in the light reflected on them from those of the drunkard and the debauchee. (Tahta, 1972, 11).

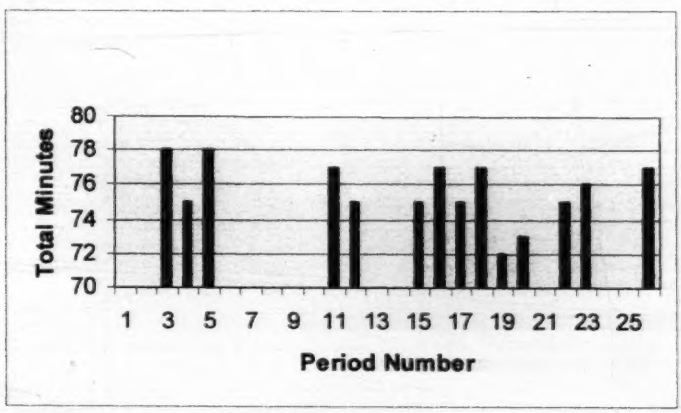
[5] Clandinin and Connelly refer to Bateson's peripheral visions instead of peripheral senses. I prefer to be reminded about how I can interact with texts intimately. For example, how I might re-visit the places of researching and writing for additional recollections.

[6] This point is not trivial. Pointing out intentions communicates a value judgement and pointing can intend various responses such as attempting to illustrate reasons for other apparently-related responses. Here, I remember trying to suspend judgements while wanting to address intentions about them.

[7] The teacher would walk around and involve herself at tables when it seemed her resources were prepared and no one was addressing her at the front of the room. While space for an extra chair does create a feeling of belonging (both made and accepted), the extra chair is not always present as a reminder. Also, the area between five seated individuals and six, separated equally, is not great enough to cause this reminder. Furthermore, the teacher does not draw attention to this as a point of welcoming and belonging.

[8] An interesting graph can be made of these beginning and ending times. A number of speculations can be made about various patterns based on the precision of recorded time. (Note: care is taken for

noting times. For example, 77 minutes and 31 seconds is rounded up to 78 minutes; and 77 minutes and 30 seconds is rounded down to 77 minutes. This seems to accommodate about 1 minute for errors and disagreements about identifying start and stop times.) For example, days three, four, and five suggest an alternating pattern to accommodate instructional time with student motivated questions, answers, and narratives outside class instructional minutes, per week. This pattern remains consistent until day sixteen, when the pattern changes from more minutes on odd days to even days (retaining a two-day cycle and after a cancelled period). Afterwards, it seems class instructional minutes are balanced with activities occurring near the end of a course requiring more period transitional time, such as difficult assignments, preparing for final assignments and exams, and ensuring curricular items are addressed.



[9] Two periods skew the total time. During these periods, numbers twenty and twenty-two, the teacher transforms the classroom into various measurement centers. The patterns of talking and interaction, and the sort of talking, change drastically. A relatively short introduction is followed by about 60 minutes of table focus for both periods. When these periods are included, the total time students focus to their tables increases from one-quarter to one-third.

[10] This description and word comes from a meeting with the teacher near the beginning of the semester.

[11] This pattern with "drama" seems a function of familiarity. I noted the same pattern during meetings with the teacher through the semester.

[12] These words from Elizabeth communicate intentions about verb tense and boundaries.

[13] Elizabeth talks about this in a variety of our meetings, by relating her classroom observations, relating research literature, and these opinions correlate with those of other writers such as Loewenberg-Ball.

[14] This comment comes from a meeting between Elizabeth and I after a period.

[15] The following is a summary image of the video, one that Elizabeth and I referenced during our interview.

> A black and white classroom image of a teenager, sitting at the front of a row, during an episode of reporting back, rolls his eyes after a classmate's blackboard turn. Adult stories accompany the narrator's conjecture: being judged as less.

[16] I was able to record two and sometimes three students at this table with videotape as I pointed the camera towards the rest of the class. The camera's microphone and another audio-tape record their words.

[17] This period is the third in a sequence of four that I did not video tape. At the time, I was attempting to verify if the video camera was motivating students to behave in certain ways; like sitting still, open postures towards the speaker, silence when the teacher speaks, silence when a student asks or answers questions, minimal whispers during any single person talking, and routines that the teacher did not initiate, such as raising hands. Behaviours among students seemed, to me, too uniform. A small lecture-type recorder was placed beside me on the side counter while I sat at a table.

[18] During interviews, Elizabeth describes various reasons. For example, questioning a student's words, by another student, can be misunderstood as a personal attack. By moderating, she ensures class discussions approach concepts like safety and respect. Also, Elizabeth does not implement structures to encourage people at different tables to talk with each other. Her intentional design about physical and social boundaries begins with how the students organize themselves for security and stability. Students will sit with those they feel comfortable. They will talk and become more familiar. If students are unfamiliar and uncomfortable with the elementary mathematics questions and activities, tensions seem cumulative. Elizabeth talks about cumulative tension as stress. Stress decreases the amount of learning. Encouraging places of familiarity and comfort are priorities for Elizabeth as she also talks about previous situations designed to encouraging socialization between tables not working well. In fact, Elizabeth addresses the emotional wellbeing of her students during the introduction of this class. She tells them that many of them are uncomfortable with elementary mathematics, invites them to reflect-on their own personal student narratives, and integrate those reflections into today's lessons.

Turn #	Person and Time	What is heard on the Audiotape
1	Elizabeth 2:56	We are going to head into exploring how children acquire understanding of number, today. And you have all kinds of stuff on the table and yes I am going to let you play with it and discover, and explore. But, just before we do that, last class I-we had had these [elementary student exemplars] and at the end of class I said to just look at them but we really didn't get a chance to score them. And, I think it is very valuable because assessment as I said is such a tricky, tricky, um, part of teaching and yet it is tricky and it is extremely important. And the assessment can make or break the instruction. Assessment can make or break kids' attitudes, in terms of how they feel about themselves as learners, and about the subject. And you know that many of you may be harboring a little bit of negative attitudes about mathematics and part of that might be how we were instructed and how we were assessed. So, it is very important as teachers that we are clear on that …
		[Elizabeth moved to a discussion about CAMP materials describing them as everything the pre-service students need to function as teachers grading assignments.]

[19] These different phrases also cue different levels of intervention. Earlier during this period, a minor disagreement occurs between two students. To disrupt this disagreement, Elizabeth starts with two phrases intended as humour, spaced with more than 5 seconds of pause. Then she uses the phrases "OK" and "it's so interesting". "All right" is Elizabeth's final phrase and the one that students respond to with silence.

[20] This quote uses the same words that another person sitting at Pat and Benjamin's table used during the period exploring the horse problem (previous chapter). Briefly, his words describe the horse problem's answer an opinionated; that the answer depends on how a person looks at the question; and, correct answers can be different numerical values. He addresses this with the teacher at his table first (more privately) and then as an additional comment after reporting back his solution and answer to the class. Elizabeth comments to him at the table (more privately) and through her comments after his reporting back, addressing the class (more publicly). Given illustrated within-table dynamics and how mathematics and mathematics education develops in this classroom, this connection is important.

[21] "John Mason [1988, p. 33] has formulated Brousseau's notion of the didactic tension as follows:

> The *more* explicit I am about the behaviour I wish my pupils to display, the more likely it is that they will display that behaviour without recourse to the understanding which the behaviour is meant to indicate; that is, the more they will take the *form* for the substance.
> The less explicit I am about my aims and expectations about the behaviour I wish my pupils to display, the less likely they are to see the point, to encounter what was intended, or to realise what it was all about." (Pimm, 1993b, 28; *emphasis in original*)

[22] "Every one of us has real difficulties getting in the way of achieving our potential as people, and the schoolchildren in our society are no exception." (Pimm, 1988, 182)

"So all the way from before the emergence of interest to the final moment of completion, there are hurdles to overcome, hurdles which your pupils unawarely carry around in their baggage and set up, unawarely, with the help of you and others. What they *are* likely to be aware of as they encounter their hurdles are feelings such as anxiety, fear, embarrassment, shame, unease, suspicion, contempt and so on. And whether the can get over those hurdles reasonably well or whether they come through limping and damaged, or whether they fall and retire feeling humiliated, or decide not to attempt them, are the factors which affect their learning and development as persons." (Pimm, 1988, 179, *emphasis in original*)

[23] Given a perimeter of 24 cm illustrate as many polygons as possible. Group your polygons according to general shape, then specific name. For each specific name, describe how the shape changes to fit the 24 cm perimeter noting important numbers and why. (Attempt graphic organizers like a chart.) As you write your answers, imagine writing this as a letter to another young mathematician.

[24] These two statements should be read considering grade-school students and post-secondary students echoing processes about a duality of curriculum (see Chapter 4).

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Appendix 1: Consent Letters

Instructor letter

The purpose of this letter is to invite your participation in a case study connected with EDEL 316, to serve as a written confirmation of our previous conversations, to serve as our written agreement of the researcher's involvement in your class, and to provide a written reminder of your voluntary consent with the research.

This case study will form the research data for my master's thesis; the endpoint of a focused year studying full time at this university in Secondary Education. My study revolves around EDEL 316 in relation to topics about math curriculum sophistication from grades 1-12, cooperative learning, and math questions utilizing group dynamics. I have always been interested with the dynamics of my own classroom; however, I have not had the time in the past to research it in depth. I know students enter with expectations and intentions about the course I teach and I, as a math teacher, have intentions and expectations. I wonder about how these match and interact as the class proceeds during the term. I also wonder about how my views of learning and teaching interact with my students' views, the particular skills necessary with different views, and how teachers and students adjust their actions to function as a class. One of my goals is to explore different teacher and student identities and describe how they interact. One of my specific questions will be how students and teachers come to define learning and teaching in a preservice education undergrad class.

My function in class will be only as an observer. I will not function as an instructor and will have no input in anyone's assessment or evaluation. Moreover, I will not be functioning as a student by participating in classroom activities. My notes and observations will relate to the research topic of intentions. However, I may sit at student tables, listen to conversations, and ask and answer questions at appropriate times during the class. Moreover, to respect your teaching role during the class, asking direct questions to you may require separate conversations. I will be expecting that you inform me of dates and times that are appropriate for you and if you will allow recording devices. I believe you will benefit by this research by having someone who is watching the occurrences of class and available for conversation. Please note that my purpose is not to offer criticism or possible solutions to issues you or I believe are happening. One of my beliefs is the occurrences of a classroom are a complex combination of inputs without simple solutions, especially considering teaching and learning. If solutions are present, these should point to a particular definition of teaching and learning. With my experiences, I will be able to offer, only if asked, my impressions about the class and contribute to our conversations in unique ways. Again, however, my first purpose in these conversations will be to understand your intentions and not to offer my experiences, methods, specifics about praxis, pedagogy, and any of the like to influence your intentions or to create tensions with our relationship.

Due to our prior conversations, I do not believe I have omitted anything. However, I do want to ensure your voluntary consent. If there is anything you would

like to add to this written agreement, please do so with the lined sheet following this letter. There will be space for both of our signatures, and my advisor's signature.

Research duration, procedures, and participation

- Duration will be the term of the class, during regular class periods and irregular meetings as required with the instructor and students. Additional meetings, as agreed to by participants, may occur with the researcher and participants.
- Observations will include my journal and field notes, video taping (audio taping), formal and informal interviews, surveys / written records as required, and materials provided by the instructor.
- With your permission and the permission of the participants, I will want to read the participants assignments. I will not be participating in the grading in any way. You are free to choose when I review any written work. If I do review any submitted work, I will not extend the time you keep the assignments from the students. Please provide for me a time you would like the package of work returned. Moreover, I will only be recording hand written summary notes and will not be making copies without additional written permission from the author.
- Any notes, video/audio tapes, and other written records will be kept in a secure place and destroyed according to university guidelines.
- The video/audio tapes will be used only for my record and remembrance. This use mimics the use employed by a regular undergrad student for study. These tapes will not be viewed by anyone other than me and my supervisor. (He is responsible for the research I conduct.)

- Any student and the instructor have the right not to participate from the start or decide to stop participating without any hesitation or question.
- If you do decide to opt out, at any time, any data collection will be withdrawn from the study.
- I will take your privacy, anonymity, and confidentiality seriously in my research. Although I may know your name it will not be recorded in my thesis. Your name will only be used to track your input to the study. In any writings, your name will be substituted by a pseudonym.
- If you choose to participate in this study, please keep a copy of this letter, sign another, and return the signed one to me. If you require additions to this letter you will be provided a photocopy, with all the additions and the required signatures.

Student letter

Welcome to EDEL 316, Communication through Mathematics. The purpose of this letter is to invite your participation in a case study occurring in this class.

This case study will form the research data for my master's thesis; the end point of a focused year studying full time at this university in Secondary Education. My study revolves around this course in relation to topics about math curriculum sophistication from grades 1-12, cooperative learning, and math questions utilizing group dynamics (variable entry prompts). I have always been interested with the dynamics of my own classroom; however, I have not had the time in the past to research it in depth. I know students enter with expectations and intentions about the course I teach and I, as a math teacher, have intentions and expectations. I wonder about how these match and interact as the class proceeds during the term. I also wonder about how my views of learning and teaching interact with my students' views, the particular skills necessary with different views, and how teachers and students adjust their actions to function as a class. One of my goals is to explore different teacher and student identities and describe how they interact. One of my specific questions will be how students and teachers come to define learning and teaching in a preservice education undergrad class.

My function in class will be only as an observer. I will not function as an instructor and will have no input in anyone's assessment or evaluation. Moreover, I will not be functioning as a student by participating in classroom activities. My notes and observations will relate to the research topic of intentions. However, I may sit at your table, listen to conversations, and ask and answer questions. If you have any

questions or concerns, please address them to me or the instructor. I believe everyone will benefit from my descriptions and questions since they deal with teaching and learning. My hope is giving everyone involved opportunities to discuss pedagogical issues, especially related to math education.

This research uses the perspective of an observer documenting the interactions between the instructor and students while asking questions about everyone's intentions and how these develop. (Please see below for specific information.) EDEL 316 is an interesting course combining, as it does, its mandatory enrolment and its mathematics content. I hope you will be willing to share your thoughts with me. I also hope this work will be published and shared in normal academic arenas (articles, presentations, teaching).

Research duration, procedures, and participation

- Duration will be the term of the class, during regular class periods and irregular meetings as required with the instructor and students. Additional meetings, as agreed to by participants, may occur with the researcher and participants.
- Observations will include my journal and field notes, video taping (audio taping), formal and informal interviews, surveys / written records as required, and materials provided by the instructor.
- With your permission and the permission of the participants, I will want to read the participants assignments. I will not discuss your assignments with your instructor. I will not be participating in the grading in any way. You are free to choose when I review any written work. If I do review any submitted work, I will not extend the time you keep the assignments from the students. Please provide for me a time you would like the package of work returned. Moreover, I will only be recording hand written summary notes and will not be making copies without additional written permission from the author.
- Any notes, video/audio tapes, and other written records will be kept in a secure place and destroyed according to university guidelines.
- The video/audio tapes will be used only for my record and remembrance. This use mimics the use employed by a regular undergrad student for study. These tapes will not be viewed by anyone other than me and my supervisor. (He is responsible for the research I conduct.)

- Any student and the instructor have the right not to participate from the start or decide to stop participating without any hesitation or question.
- If you do decide to opt out, at any time, any data collection will be withdrawn from the study.
- I will take your privacy, anonymity, and confidentiality seriously in my research. Although I may know your name it will not be recorded in my thesis. Your name will only be used to track your input to the study. In any writings, your name will be substituted by a pseudonym.
- If you choose to participate in this study, please keep a copy of this letter, sign another, and return the signed one to me. If you require additions to this letter you will be provided a photocopy, with all the additions and the required signatures.